Math S-21b – Summer 2004 – Practice Exam #2

1) TRUE/FALSE (circle one)

a) Orthogonal projection onto a subspace is an orthogonal transformation.	True False
b) If **A** is any square matrix, then det (–**A**) = – det(**A**).	True False
c) If **A** is an invertible matrix that is <u>similar</u> to its own inverse $\mathbf{A}^{-1}$, then det(**A**) must be either +1 or –1.	True False
d) Let **A** be an orthogonal matrix. Then det(**A**) = 1.	True False
e) If **S** and **A** are orthogonal $n \times n$ matrices, then the matrix $\mathbf{S}^{-1}\mathbf{AS}$ is orthogonal as well.	True False
f) It is always the case that $\det(\mathbf{A}^T\mathbf{A}) \geq 0$.	True False
g) If λ is an eigenvalue of **A** with eigenvector **v**, μ is a distinct eigenvalue with eigenvector **w**, then **v** + **w** is also an eigenvector of **A**.	True False
h) If **A** is an n × n matrix, then **A** and $\mathbf{A}^T$ have the same eigenvalues.	True False

2) Let P_2 is the linear space consisting of all polynomials of degree ≤ 2, and let $T: P_2 \to P_2$ be defined by

$$T(f) = f'' - 2f' + 3f\ .$$

a) Find the matrix of this linear transformation relative to the basis $\{1, t, t^2\}$.
b) Find the determinant of this linear transformation.
c) Is this linear transformation invertible, i.e. an isomorphism? Briefly explain.
d) Using the above information, find a quadratic polynomial $f(t)$ such that $f'' - 2f' + 3f = 3t^2$.

3) Consider the following <u>inconsistent</u> system of linear equations: $\begin{Bmatrix} 2x + y = 2 \\ x - y = 1 \\ x + 2y = -2 \end{Bmatrix}$.

a) Find the least-squares solution for this linear system. [*Note*: This is <u>not</u> a data-fitting problem.]
b) Each equation in the given system represents a line in $\mathbf{R}^2$. Describe in words and/or pictures the relationship between these three lines and the point you found in part a).

4) We are given three vectors in $\mathbf{R}^4$: $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$.

a) Find the area of the parallelogram determined by the vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$.
b) Construct an <u>orthonormal basis</u> for the two-dimensional subspace of $\mathbf{R}^4$ spanned by $\{\mathbf{v}_1, \mathbf{v}_2\}$. Call the vectors of this orthonormal basis $\mathbf{w}_1$ and $\mathbf{w}_2$.
c) Find the orthogonal projection of $\mathbf{v}_3$ in the subspace spanned by the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$.
d) If we let $\mathbf{B} = \begin{bmatrix} \uparrow & \uparrow \\ \mathbf{w}_1 & \mathbf{w}_2 \\ \downarrow & \downarrow \end{bmatrix}$ where $\{\mathbf{w}_1, \mathbf{w}_2\}$ is the orthonormal basis found in part b,
what are the values of $\det(\mathbf{BB}^T)$ and $\det(\mathbf{B}^T\mathbf{B})$?
[You don't need to know what $\mathbf{w}_1$ and $\mathbf{w}_2$ are to calculate these two numbers.]