

Chapter 1

1.1

$$1. \begin{array}{l} \left| \begin{array}{l} x + 2y = 1 \\ 2x + 3y = 1 \end{array} \right| \xrightarrow{-2 \times \text{2nd equation}} \left| \begin{array}{l} x + 2y = 1 \\ -y = -1 \end{array} \right| \xrightarrow{\div(-1)} \\ \left| \begin{array}{l} x + 2y = 1 \\ y = 1 \end{array} \right| \xrightarrow{-2 \times \text{2nd equation}} \left| \begin{array}{l} x = -1 \\ y = 1 \end{array} \right|, \text{ so that } (x, y) = (-1, 1). \end{array}$$

$$2. \begin{array}{l} \left| \begin{array}{l} 4x + 3y = 2 \\ 7x + 5y = 3 \end{array} \right| \xrightarrow{\div 4} \left| \begin{array}{l} x + \frac{3}{4}y = \frac{1}{2} \\ 7x + 5y = 3 \end{array} \right| \xrightarrow{-7 \times \text{1st equation}} \\ \left| \begin{array}{l} x + \frac{3}{4}y = \frac{1}{2} \\ -\frac{1}{4}y = -\frac{1}{2} \end{array} \right| \xrightarrow{\times(-4)} \left| \begin{array}{l} x + \frac{3}{4}y = \frac{1}{2} \\ y = 2 \end{array} \right| \xrightarrow{-\frac{3}{4} \times \text{2nd equation}} \left| \begin{array}{l} x = -1 \\ y = 2 \end{array} \right|, \text{ so that } (x, y) = (-1, 2). \end{array}$$

$$3. \left| \begin{array}{l} 2x + 4y = 3 \\ 3x + 6y = 2 \end{array} \right| \xrightarrow{\div 2} \left| \begin{array}{l} x + 2y = \frac{3}{2} \\ 3x + 6y = 2 \end{array} \right| \xrightarrow{-3 \times \text{1st equation}} \left| \begin{array}{l} x + 2y = \frac{3}{2} \\ 0 = -\frac{5}{2} \end{array} \right|$$

There is no solution.

$$4. \left| \begin{array}{l} 2x + 4y = 2 \\ 3x + 6y = 3 \end{array} \right| \xrightarrow{\div 2} \left| \begin{array}{l} x + 2y = 1 \\ 3x + 6y = 3 \end{array} \right| \xrightarrow{-3 \times \text{1st equation}} \left| \begin{array}{l} x + 2y = 1 \\ 0 = 0 \end{array} \right|$$

This system has infinitely many solutions: if we choose $y = t$, an arbitrary real number, then the equation $x + 2y = 1$ gives us $x = 1 - 2y = 1 - 2t$. Therefore the general solution is $(x, y) = (1 - 2t, t)$, where t is an arbitrary real number.

$$5. \begin{array}{l} \left| \begin{array}{l} 2x + 3y = 0 \\ 4x + 5y = 0 \end{array} \right| \xrightarrow{\div 2} \left| \begin{array}{l} x + \frac{3}{2}y = 0 \\ 4x + 5y = 0 \end{array} \right| \xrightarrow{-4 \times \text{1st equation}} \\ \left| \begin{array}{l} x + \frac{3}{2}y = 0 \\ -y = 0 \end{array} \right| \xrightarrow{\div(-1)} \left| \begin{array}{l} x + \frac{3}{2}y = 0 \\ y = 0 \end{array} \right| \xrightarrow{-\frac{3}{2} \times \text{2nd equation}} \left| \begin{array}{l} x = 0 \\ y = 0 \end{array} \right|, \text{ so that } (x, y) = (0, 0). \end{array}$$

$$6. \begin{array}{l} \left| \begin{array}{l} x + 2y + 3z = 8 \\ x + 3y + 3z = 10 \\ x + 2y + 4z = 9 \end{array} \right| \xrightarrow{\begin{array}{l} -I \\ -I \end{array}} \left| \begin{array}{l} x + 2y + 3z = 8 \\ y = 2 \\ z = 1 \end{array} \right| \xrightarrow{-2(II)} \\ \left| \begin{array}{l} x + 3z = 4 \\ y = 2 \\ z = 1 \end{array} \right| \xrightarrow{-3(III)} \left| \begin{array}{l} x = 1 \\ y = 2 \\ z = 1 \end{array} \right|, \text{ so that } (x, y, z) = (1, 2, 1). \end{array}$$

$$7. \left| \begin{array}{l} x + 2y + 3z = 1 \\ x + 3y + 4z = 3 \\ x + 4y + 5z = 4 \end{array} \right| \xrightarrow{\begin{array}{l} -I \\ -I \end{array}} \left| \begin{array}{l} x + 2y + 3z = 1 \\ y + z = 2 \\ 2y + 2z = 3 \end{array} \right| \xrightarrow{\begin{array}{l} -2(II) \\ -2(II) \end{array}} \left| \begin{array}{l} x + z = -3 \\ y + z = 2 \\ 0 = -1 \end{array} \right|$$

This system has no solution.

15. The system reduces to $\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$ so the unique solution is $(x, y, z) = (0, 0, 0)$. The three planes intersect at the origin.

16. The system reduces to $\begin{cases} x + 5z = 0 \\ y - z = 0 \\ 0 = 0 \end{cases}$, so the solutions are of the form $(x, y, z) = (-5t, t, t)$, where t is an arbitrary number. The three planes intersect in a line; compare with Figure 2a.

17. $\begin{cases} x + 2y = a \\ 3x + 5y = b \end{cases} \xrightarrow{-3(I)} \begin{cases} x + 2y = a \\ -y = -3a + b \end{cases} \xrightarrow{\div(-1)} \begin{cases} x + 2y = a \\ y = 3a - b \end{cases} \xrightarrow{-2(II)} \begin{cases} x = -5a + 2b \\ y = 3a - b \end{cases}$, so that $(x, y) = (-5a + 2b, 3a - b)$.

29. To assure that the graph goes through the point $(1, -1)$, we substitute $t = 1$ and $f(t) = -1$ into the equation $f(t) = a + bt + ct^2$ to give $-1 = a + b + c$.

Proceeding likewise for the two other points, we obtain the system
$$\begin{cases} a + b + c = -1 \\ a + 2b + 4c = 3 \\ a + 3b + 9c = 13 \end{cases}.$$

The solution is $a = 1$, $b = -5$, and $c = 3$, and the polynomial is $f(t) = 1 - 5t + 3t^2$.

