5. $$\left[\begin{array}{cccc:c} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{I \leftrightarrow III} \left[\begin{array}{cccc:c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} \\ \longrightarrow \\ \\ -(I) \end{array} \left[\begin{array}{cccc:c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} -II \\ \longrightarrow \\ \\ +II \end{array}$$

$$\left[\begin{array}{cccc:c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{c} +III \\ \xrightarrow{-III} \\ \\ -III \end{array} \left[\begin{array}{cccc:c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left|\begin{array}{ccccccc} x_1 & & & + & x_4 & = & 0 \\ & x_2 & & - & x_4 & = & 0 \\ & & x_3 & + & x_4 & = & 0 \end{array}\right| \longrightarrow \left|\begin{array}{l} x_1 = -x_4 \\ x_2 = x_4 \\ x_3 = -x_4 \end{array}\right|$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ -t \\ t \end{bmatrix},$$ where t is an arbitrary real number.

10. The system reduces to $\left|\begin{array}{ccccccc} x_1 & & & + & x_4 & = & 1 \\ & x_2 & & - & 3x_4 & = & 2 \\ & & x_3 & + & 2x_4 & = & -3 \end{array}\right| \longrightarrow \left|\begin{array}{l} x_1 = 1 - x_4 \\ x_2 = 2 + 3x_4 \\ x_3 = -3 - 2x_4 \end{array}\right|$

Let $x_4 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 - t \\ 2 + 3t \\ -3 - 2t \\ t \end{bmatrix}$$, where t is an arbitrary real number.

11. The system reduces to $\left|\begin{array}{cccccc} x_1 & & + & 2x_3 & & = & 0 \\ & x_2 & - & 3x_3 & & = & 4 \\ & & & & x_4 & = & -2 \end{array}\right| \longrightarrow \left|\begin{array}{l} x_1 = -2x_3 \\ x_2 = 4 + 3x_3 \\ x_4 = -2 \end{array}\right|.$

Let $x_3 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2t \\ 4 + 3t \\ t \\ -2 \end{bmatrix}$$

16. The system reduces to $\left|\begin{array}{lll} x_1 + 2x_2 + 3x_3 \quad\quad +5x_5 & = & 6 \\ \quad\quad\quad\quad\quad x_4 \quad +2x_5 & = & 7 \end{array}\right| \longrightarrow \left|\begin{array}{l} x_1 = 6 - 2x_2 - 3x_3 - 5x_5 \\ x_4 = 7 - 2x_5 \end{array}\right|$.

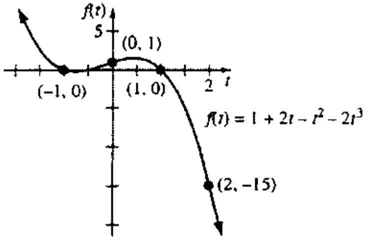
Let $x_2 = r$, $x_3 = s$, and $x_5 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 6 - 2r - 3s - 5t \\ r \\ s \\ 7 - 2t \\ t \end{bmatrix}$$

17. The system reduces to $\left|\begin{array}{lcr} x_1 & = & -\dfrac{8221}{4340} \\ \quad x_2 & = & \dfrac{8591}{8680} \\ \quad\quad x_3 & = & \dfrac{4695}{434} \\ \quad\quad\quad x_4 & = & -\dfrac{459}{434} \\ \quad\quad\quad\quad x_5 & = & \dfrac{699}{434} \end{array}\right|$

30. Plugging the points into $f(t)$, we obtain the system $\left| \begin{array}{ccccccccc} a & & & & & & & = & 1 \\ a & + & b & + & c & + & d & = & 0 \\ a & - & b & + & c & - & d & = & 0 \\ a & + & 2b & + & 4c & + & 8d & = & -15 \end{array} \right|$

with unique solution $a = 1,\ b = 2,\ c = -1$, and $d = -2$, so that $f(t) = 1 + 2t - t^2 - 2t^3$.



34. We want all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ such that $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = x + 3y - z = 0$. The endpoints of these vectors form a plane.

These vectors are of the form $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3r + t \\ r \\ t \end{bmatrix}$, where r and t are arbitrary real numbers.

35. We need to solve the system $\left| \begin{array}{r} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ x_1 + 9x_2 + 9x_3 + 7x_4 = 0 \end{array} \right|$,

which reduces to $\left| \begin{array}{ccccccc} x_1 & & & + & 0.25x_4 & = & 0 \\ & x_2 & & - & 1.5x_4 & = & 0 \\ & & x_3 & + & 2.25x_4 & = & 0 \end{array} \right|$

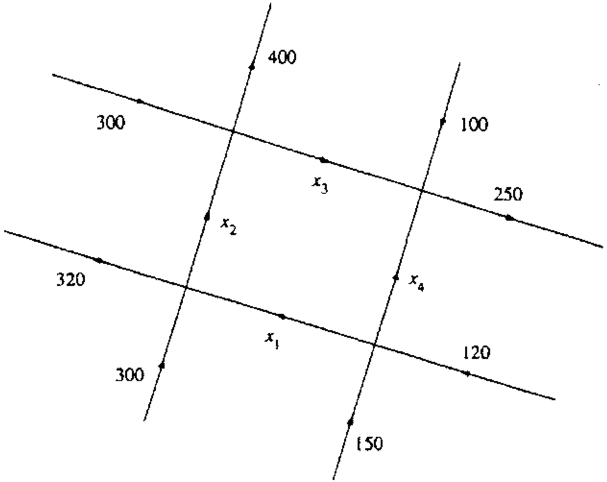
The solutions are of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -0.25t \\ 1.5t \\ -2.25t \\ t \end{bmatrix}$, where t is an arbitrary real number.

36. Writing the equation $\vec{b} = x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3$ in terms of its components, we obtain the system

$$\left| \begin{array}{rcr} x_1 + 2x_2 + 4x_3 & = & -8 \\ 4x_1 + 5x_2 + 6x_3 & = & -1 \\ 7x_1 + 8x_2 + 9x_3 & = & 9 \\ 5x_1 + 3x_2 + x_3 & = & 15 \end{array} \right|$$

The system has the unique solution $x_1 = 2$, $x_2 = 3$, and $x_3 = -4$.

42. Let x_1, x_2, x_3, and x_4 be the traffic volume at the four locations indicated below.



We are told that the number of cars coming into each intersection is the same as the number of cars coming out:

$$\left| \begin{array}{r} x_1 + 300 = 320 + x_2 \\ x_2 + 300 = 400 + x_3 \\ x_3 + x_4 + 100 = 250 \\ 150 + 120 = x_1 + x_4 \end{array} \right| \text{ or } \left| \begin{array}{ccccccccc} x_1 & - & x_2 & & & & & = & 20 \\ & & x_2 & - & x_3 & & & = & 100 \\ & & & & x_3 & + & x_4 & = & 150 \\ x_1 & & & & & + & x_4 & = & 270 \end{array} \right|$$

The solutions are of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 270 - t \\ 250 - t \\ 150 - t \\ t \end{bmatrix}$.

Since the x_i must be positive integers (or zero), t must be an integer with $0 \le t \le 150$.
The lowest possible values are $x_1 = 120$, $x_2 = 100$, $x_3 = 0$, and $x_4 = 0$, while the highest possible values are $x_1 = 270$, $x_2 = 250$, $x_3 = 150$, and $x_4 = 150$.