

## 2.3

1.  $\text{rref} \begin{bmatrix} 2 & 3 & 1 & 0 \\ 5 & 8 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 8 & -3 \\ 0 & 1 & -5 & 2 \end{bmatrix}$ , so that  $\begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix}^{-1} = \begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix}$ .

2.  $\text{rref} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ , so that  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$  is not invertible.

3.  $\text{rref} \begin{bmatrix} 0 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} & 0 \end{bmatrix}$ , so that  $\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix}$ .

4. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} \frac{3}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{3}{2} & 1 & \frac{1}{2} \end{bmatrix}$ .

5.  $\text{rref} \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ , so that the matrix is not invertible, by Fact 2.3.3.

6. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ .

7.  $\text{rref} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ , so that the matrix is not invertible, by Fact 2.3.3.

8. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ .

9.  $\text{rref} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ , so that the matrix is not invertible, by Fact 2.3.3.

10. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$ .

11. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .

12. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} 5 & -20 & -2 & -7 \\ 0 & -1 & 0 & 0 \\ -2 & 6 & 1 & 2 \\ 0 & 3 & 0 & 1 \end{bmatrix}$

13. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix}$ .

14. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} 3 & -5 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 5 & -2 \\ 0 & 0 & -2 & 1 \end{bmatrix}$ .

15. Use Fact 2.3.5; the inverse is  $\begin{bmatrix} -6 & 9 & -5 & 1 \\ 9 & -1 & -5 & 2 \\ -5 & -5 & 9 & -3 \\ 1 & 2 & -3 & 1 \end{bmatrix}$

41. a. Invertible: the transformation is its own inverse.

- b. Not invertible: the equation  $T(\vec{x}) = \vec{b}$  has infinitely many solutions if  $\vec{b}$  is on the plane, and none otherwise.
- c. Invertible: The inverse is dilation by  $\frac{1}{5}$  (that is, a contraction by 5). If  $\vec{y} = 5\vec{x}$ , then  $\vec{x} = \frac{1}{5}\vec{y}$ .
- d. Invertible: The inverse is a rotation about the same axis through the same angle in the opposite direction.