8. We need to solve the system $c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

The general solution is $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} t \\ -2t \\ t \end{bmatrix}$.

Picking $t = 1$ we find the nontrivial relation $1\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

9. These vectors are linearly dependent, since $\vec{v}_m = 0\vec{v}_1 + 0\vec{v}_2 + \cdots + 0\vec{v}_{m-1}$.

10. Linearly dependent, since $\begin{bmatrix} 6 \\ 3 \end{bmatrix} = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

11. Linearly independent, since the two vectors are not parallel.

12. Linearly dependent, since $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\begin{bmatrix} 7 \\ 11 \end{bmatrix}$.

13. Linearly dependent, since $\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

14. Linearly independent, since ref $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} = I_3$ (use Fact 3.2.6).

15. Linearly dependent, since ker $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \neq \{\vec{0}\}$ (by Fact 1.3.3).

16. Linearly dependent, since rref $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ (use Fact 3.2.6).

17. Linearly independent, since rref $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} = I_3$ (use Fact 3.2.6).

18. Linearly dependent, since rref $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \\ 1 & 4 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

19. Linearly dependent, since ker $\begin{bmatrix} 1 & 1 & 5 & 1 & 1 \\ 2 & 5 & 4 & 8 & 1 \\ 9 & 1 & 9 & 1 & 1 \\ 1 & 5 & 1 & 5 & 1 \end{bmatrix} \neq \{\vec{0}\}$ (by Fact 1.3.3).

20. Linearly dependent, since rref $\begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

21. Linearly dependent, since rref does not have a leading one in the fourth column.

22. If a, c and f are nonzero, then $\text{rref}\begin{bmatrix} a & b & d \\ 0 & c & e \\ 0 & 0 & f \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, and the three vectors are linearly independent, by Fact 3.2.6. If at least one of the constants a, c or f is zero, then at least one column of rref will not contain a leading one, so that the three vectors are linearly dependent.

23. The zero vector is in $V^\perp$, since $\vec{0} \cdot \vec{v} = 0$ for all $\vec{v}$ in V.

If $\vec{w}_1$ and $\vec{w}_2$ are both in $V^\perp$, then $(\vec{w}_1 + \vec{w}_2) \cdot \vec{v} = \vec{w}_1 \cdot \vec{v} + \vec{w}_2 \cdot \vec{v} = 0 + 0 = 0$ for all $\vec{v}$ in V, so that $\vec{w}_1 + \vec{w}_2$ is in $V^\perp$ as well.

If $\vec{w}$ is in $V^\perp$ and k is an arbitrary constant, then $(k\vec{w}) \cdot \vec{v} = k(\vec{w} \cdot \vec{v}) = k0 = 0$ for all $\vec{v}$ in V, so that $k\vec{w}$ is in $V^\perp$ as well.

24. We need to find all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ such that $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = x + 2y + 3z = 0$.

These vectors have the form $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2s - 3t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$.

Therefore, $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ is a basis of $L^\perp$.