7. $\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = 0$

8. $\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \end{bmatrix} = 6$

9. $\det \begin{bmatrix} 7 & 8 & 9 \\ 0 & 0 & 0 \\ 5 & 6 & 7 \end{bmatrix} = 0$

15. $\det \begin{bmatrix} 0 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 4 \end{bmatrix} = 24$

16. $\det \begin{bmatrix} 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 9 & 7 & 9 & 3 \\ 0 & 0 & 0 & 0 & 5 \\ 3 & 4 & 5 & 8 & 5 \end{bmatrix} = 540$

26. We will compute $\det(\lambda I_2 - A) = \det \begin{bmatrix} \lambda - 1 & -3 \\ 0 & \lambda - 3 \end{bmatrix} = (\lambda - 1)(\lambda - 3)$, so the matrix is not invertible when $(\lambda - 1)(\lambda - 3) = 0$, i.e., when $\lambda = 1$ or $\lambda = 3$.

27. We will compute $\det(\lambda I_2 - A) = \det \begin{bmatrix} \lambda - 4 & -2 \\ -2 & \lambda - 7 \end{bmatrix} = \lambda^2 - 11\lambda + 24 = (\lambda - 3)(\lambda - 8)$, so the matrix is not invertible when $(\lambda - 3)(\lambda - 8) = 0$, i.e., when $\lambda = 3$ or $\lambda = 8$.

28. We will compute $\det(\lambda I_2 - A) = \det \begin{bmatrix} \lambda - 2 & 3 \\ -3 & \lambda - 2 \end{bmatrix} = \lambda^2 + 4\lambda + 13$. Since $\lambda^2 - 4\lambda + 13 = 0$ has no real solutions, the matrix is invertible for all real values of λ.

29. We will compute $\det(\lambda I_2 - A) = \det \begin{bmatrix} \lambda - 3 & -1 \\ 4 & \lambda + 1 \end{bmatrix} = (\lambda + 1)^2$, so the matrix is not invertible when $(\lambda - 1)^2 = 0$, i.e., when $\lambda = 1$.

30. We will compute $\det(\lambda I_3 - A) = (\lambda - 1)(\lambda - 2)(\lambda - 3)$. The matrix is not invertible when $(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$, i.e., when $\lambda = 1$ or $\lambda = 2$ or $\lambda = 3$.

31. We will compute $\det(\lambda I_3 - A) = \det \begin{bmatrix} \lambda & 0 & -1 \\ 0 & \lambda & -1 \\ -1 & -1 & \lambda - 1 \end{bmatrix} = \lambda(\lambda - 2)(\lambda + 1)$, so the matrix is not invertible when $\lambda(\lambda - 2)(\lambda + 1) = 0$, i.e., when $\lambda = 0, \lambda = 2$, or $\lambda = -1$.

32. We will compute $\det(\lambda I_3 - A) = \det \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 0 & -8 & \lambda + 2 \end{bmatrix} = \lambda(\lambda + 4)(\lambda - 2)$, so the matrix is not invertible when $\lambda(\lambda + 4)(\lambda - 2) = 0$, i.e. when $\lambda = 0$, $\lambda = -4$, or $\lambda = 2$.

46. The work on page 243 in the text shows that $\det[\vec{u}\,\vec{v}\,\vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$, for any 3×3 matrix $[\vec{u}\,\vec{v}\,\vec{w}]$.

Therefore, $\det(A) = \det[\vec{v} \times \vec{w}\ \vec{v}\ \vec{w}] = (\vec{v} \times \vec{w}) \cdot (\vec{v} \times \vec{w}) = \|\vec{v} \times \vec{w}\|^2$.

The matrix A is invertible if $\vec{v} \times \vec{w} \neq 0$, that is, if $\vec{v}$ and $\vec{w}$ are not parallel.

47. Consider a pattern in A, with entries $a_1, \ldots, a_n$; the corresponding pattern in $-A$ will have the entries $-a_1, \ldots, -a_n$. The products associated with these patterns will be $a_1 a_2 \cdots a_n$ and $(-1)^n a_1 a_2 \cdots a_n$, respectively. Since these observations apply to all patterns, we can say that $\det(-A) = (-1)^n \det(A)$.

48. $\det(kA) = k^n \det(A)$
The argument is analogous to the one in Exercise 47.