5. Assume $A\vec{v} = \lambda\vec{v}$ and $B\vec{v} = \beta\vec{v}$ for some eigenvalues λ, β. Then $(A+B)\vec{v} = A\vec{v} + B\vec{v} = \lambda\vec{v} + \beta\vec{v} = (\lambda + \beta)\vec{v}$ so $\vec{v}$ is an eigenvector of $A + B$ with eigenvalue $\lambda + \beta$.

6. Yes. If $A\vec{v} = \lambda\vec{v}$ and $B\vec{v} = \mu\vec{v}$, then $AB\vec{v} = A(\mu\vec{v}) = \mu(A\vec{v}) = \mu\lambda\vec{v}$

15. Any vector on L is unaffected by the reflection, so that a nonzero vector on L is an eigenvector with eigenvalue 1. Any vector on $L^{\perp}$ is flipped about L, so that a nonzero vector on $L^{\perp}$ is an eigenvector with eigenvalue -1. Picking a nonzero vector from L and one from $L^{\perp}$, we obtain a basis consisting of eigenvectors.

16. Rotation by 180° is a flip about the origin so every nonzero vector is an eigenvector with the eigenvalue -1. Any basis for $\mathbb{R}^2$ consists of eigenvectors.

17. No (real) eigenvalues

18. Any nonzero vector in the plane is unchanged, hence is an eigenvector with the eigenvalue 1. Since any nonzero vector in $E^{\perp}$ is flipped about the origin, it is an eigenvector with eigenvalue -1. Pick any two non-collinear vectors from E and one from $E^{\perp}$ to form a basis consisting of eigenvectors.

19. Any nonzero vector in L is an eigenvector with eigenvalue 1, and any nonzero vector in the plane $L^{\perp}$ is an eigenvector with eigenvalue 0. Form an eigenbasis by picking any nonzero vector in L and any two noncollinear vectors in $L^{\perp}$.

20. Any nonzero vector along the $\vec{e}_3$-axis is unchanged, hence is an eigenvector with eigenvalue 1. No other (real) eigenvalues can be found.

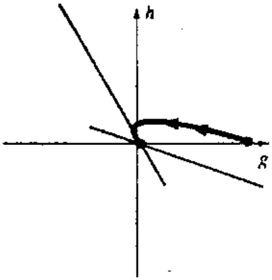
21. Any nonzero vector in $\mathbb{R}^3$ is an eigenvector with eigenvalue 5. Any basis for $\mathbb{R}^3$ consists of eigenvectors.

34. We want A such that $A\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$ and $A\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}10\\20\end{bmatrix}$, i.e. $A\begin{bmatrix}3&1\\1&2\end{bmatrix}=\begin{bmatrix}15&10\\5&20\end{bmatrix}$, so

$$A=\begin{bmatrix}15&10\\5&20\end{bmatrix}\begin{bmatrix}3&1\\1&2\end{bmatrix}^{-1}=\begin{bmatrix}4&3\\-2&11\end{bmatrix}.$$

36. a. $\begin{bmatrix} 0.978 & -0.006 \\ 0.004 & 0.992 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.99 \\ 1.98 \end{bmatrix} = 0.99 \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 0.978 & -0.006 \\ 0.004 & 0.992 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 2.94 \\ -0.98 \end{bmatrix} = 0.98 \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. The eigenvalues are $\lambda_1 = 0.99$ and $\lambda_2 = 0.98$.

b. $\vec{x}_0 = \begin{bmatrix} g_0 \\ l_0 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix} = 20 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + 40 \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ so $\vec{x}(t) = 20(0.99)^t \begin{bmatrix} -1 \\ 2 \end{bmatrix} + 40(0.98)^t \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, hence $g(t) = -20(0.99)^t + 120(0.98)^t$ and $h(t) = 40(0.99)^t - 40(0.98)^t$.



$h(t)$ first rises, then falls back to zero. $g(t)$ falls a little below zero, then goes back up to zero.

c. We set $g(t) = -20(0.99)^t + 120(0.98)^t = 0$.
Solving for t we get that $g(t) = 0$ for $t \approx 176$ minutes. (After $t = 176$, $g(t) < 0$).

39. Let λ be an eigenvalue of $S^{-1}AS$. Then for some nonzero vector $\vec{v}$, $S^{-1}AS\vec{v} = \lambda\vec{v}$, i.e., $AS\vec{v} = S\lambda\vec{v} = \lambda S\vec{v}$ so λ is an eigenvalue of A with eigenvector $S\vec{v}$.
Conversely, if α is an eigenvalue of A with eigenvector $\vec{w}$, then $A\vec{w} = \alpha\vec{w}$.
Therefore, $S^{-1}AS(S^{-1}\vec{w}) = S^{-1}A\vec{w} = S^{-1}\alpha\vec{w} = \alpha S^{-1}\vec{w}$, so $S^{-1}\vec{w}$ is an eigenvector of $S^{-1}AS$ with eigenvalue α.