Math S-21b – Summer 2005 – Practice Exam #1

(1) True or False. (Circle one) You need not give your reasoning.

a) If T_1, T_2 are two linear transformations from $\mathbf{R}^3$ to $\mathbf{R}^3$ such that $\ker(T_1) = \ker(T_2)$ and $\text{image}(T_1) = \text{image}(T_2)$, then $T_1 = T_2$.	a) TRUE FALSE
b) The kernel of rref(**A**) is the same as the kernel of **A**.	b) TRUE FALSE
c) The image of rref(**A**) is the same as the image of **A**.	c) TRUE FALSE
d) If **A** and **B** are $n \times n$ matrices such that the kernel of **A** is contained in the image of **B**, then the matrix **AB** cannot be invertible.	d) TRUE FALSE
e) Let **A** and **B** be $n \times n$ matrices, with $\mathbf{AB} = \mathbf{BA}$. Then $\mathbf{A}^3\mathbf{B} = \mathbf{BA}^3$.	e) TRUE FALSE
f) If **A** is an $n \times n$ matrix, $\mathbf{A}^2 = \mathbf{A}$, and rank(**A**) = n, then $\mathbf{A} = \mathbf{I}_n$.	f) TRUE FALSE

2) Short answer questions:

a) If $\mathbf{B} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 4 & 2 \\ 1 & 0 \end{bmatrix}$ and **AB = C**, what is **A**?

b) Let $\mathbf{A} = \begin{bmatrix} 5 & -12 \\ 12 & 5 \end{bmatrix}$.

Describe briefly, in geometric terms, the linear transformation represented by this matrix.

c) For which choices of the constant k is the matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{bmatrix}$ not invertible? Explain briefly.

d) Find a basis for the subspace of $\mathbf{R}^4$ that consists of all vectors orthogonal (perpendicular) to both $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$.

3) Let **A** be the 3 x 5 matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ -1 & 3 & -5 & 2 & 8 \\ 2 & 3 & 1 & 2 & 5 \end{bmatrix}$.

a) Find a basis for the kernel of **A** and its dimension, i.e. the nullity.
b) Find a basis for the image of **A** and its dimension, i.e. the rank.
c) Find all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}$.

4) Let $\mathcal{B}$ be the basis of $\mathbf{R}^3$ consisting of the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

Let $T: \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation such that $T(\mathbf{v}_1) = \mathbf{v}_2$, $T(\mathbf{v}_2) = \mathbf{v}_1$, and $T(\mathbf{v}_3) = -\mathbf{v}_3$.

a) Find the coordinates of the vector $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ relative to the basis $\mathcal{B}$.

b) Find the matrix **B** of T with respect to the basis $\mathcal{B}$.

c) Find the matrix **A** of T with respect to the standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ of $\mathbf{R}^3$.