Math S-21b: Linear Algebra and Differential Equations - Summer 2004

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Class meetings: Daily, 11am to noon, in Sci Ctr 110, from Mon, June 28 through Fri, Aug 13 excluding Mon, July 5. In addition, there will be an optional problem session on Tuesdays and Thursdays at 2:15pm in Sci Ctr 411 conducted by course assistant Peter Bierhorst.

Office Hours: Daily, Monday-Thursday following class, either in SC110 or SC333i.

Prerequisites: Math 1b or equivalent knowledge of algebra and calculus; and Math 21a or equivalent. You should be able to solve simple systems of equations, find the zeros of polynomials, and ideally be able to set up and solve the kind of simple differential equations found in a typical first-year calculus course. Math 21a or an equivalent multivariable calculus course is listed as a prerequisite but is not absolutely necessary for successful completion of this course and may be waived with permission of the instructor. Math 21a and Math 21b may be taken simultaneously, but doing the homework and exam preparation for both courses will require a considerable commitment of time and attention.

Philosophy: This course is greatly dependent upon your participation. Most of the mathematical concepts and techniques will be presented in class, with plenty of opportunity for questions and clarification. Please take advantage of these opportunities. The best lessons learned are those derived from discussion and practice.

Outside of class, it is essential that you do the assigned homework to learn the material. This will be the best preparation for interaction in the classroom and for the exams. Our course assistant will hold sections to clarify and expand upon material presented in class, especially the practical techniques and insights that help with the homework, and will allow a substantial fraction of time for your questions.

Homework: There will be homework assignments approximately three times per week that will be due 2-3 days later. All decisions regarding assignments turned in late will rest with the course assistant. Homework and exam solutions will be posted on the course website as PDF documents which can be read using Adobe Acrobat. All submitted homework must be neat, stapled, and answers boxed where appropriate. All assignments and solutions will be posted or linked in the **Calendar**.

Please note that the reading assigned with each homework is essential. Some topics not covered fully in class will be left to the reading and you will be expected to pick up those additional details.

Grading: There will be two hour exams and a three-hour final exam at the end of the course. As of this writing, the course grade will be computed according to:

.20(hour exam 1) + .20(hour exam 2) + .25(homework) + .35(final exam)

This scheme may change slightly.

Exam dates: First Hour Exam: Wed, July 14 in class Second Hour Exam: Fri, July 30 in class
Final Exam: Wed, August 18, 9am

Text: *Linear Algebra With Applications, Second Edition*, by Otto Bretscher, Prentice-Hall, 2001; ISBN 0-13-019857-9. A student solutions manual is also available with the text. We will cover selected topics from this book, and homework will be assigned from its large collection of exercises. Additional supplements on various topics in differential equations will be distributed.

Calculators, computers: Though not required for the course, you may find it useful to have a matrix-capable calculator such as a TI-85 or higher, especially for row reduction, matrix products, matrix inversion, and the calculation of determinants. Unless directed otherwise, calculators will be permitted on the exams.

Math S-21b course website: http://math.harvard.edu/~rwinters/mathS21b

Mathematics S-21b syllabus (may change slightly)

Date	Text sections	Topics
Week 1	1.1: Introduction to Linear Systems 1.2: Matrices and Gauss-Jordan Elimination 1.3: On the Solutions of Linear Systems 2.1: Introduction to Linear Transformations and their Inverses 2.2: Linear Transformations in Geometry 2.3: The Inverse of a Linear Transformation	Algebra and geometry of lines, planes; solving equations simultaneously; row reduction and row operations; rank of a matrix; homogeneous vs. inhomogeneous systems; Inverse of a matrix; linear transformations from $\mathbf{R}^m$ to $\mathbf{R}^n$; linearity; domain and codomain; invertibility; meaning of the columns of a matrix; rotations and dilations; shears; projections and reflections;
Week 2	2.4: Matrix Products 3.1: Image and Kernel of a Linear Transformation 3.2: Subspaces of $\mathbf{R}^n$; Bases and Linear Independence 3.3: The Dimension of a Subspace of $\mathbf{R}^n$ 3.4: Coordinates 4.1: Introduction to Linear Spaces	Matrix algebra, associativity and the composition of linear functions. Image and kernel of a linear transformation; linear combinations and the span of a set of vectors; subspaces; linear independence; basis; dimension of a subspace; bases for kernels and images; Rank-Nullity Theorem; coordinates of a vector relative to a basis; matrix of a linear transformation relative to a (nonstandard) basis; Examples of linear spaces other than $\mathbf{R}^n$, e.g. function spaces.
Week 3	4.2: Linear Transformations and Isomorphisms 4.3: Coordinates in a Linear Space **First hour exam** (Wed, July 14) 5.1: Orthogonal Bases and Orthogonal Projections 5.2: Gram-Schmidt Process and QR Factorization 5.3: Orthogonal Transformations and Orthogonal Matrices	Linear spaces; isomorphisms; coordinates; matrix of a general linear transformation relative to a basis; Orthogonality (perpendicularity) of vectors in $\mathbf{R}^n$; length (norm) of a vector, unit vectors; orthogonal complements; orthogonal projections; orthonormal basis; angle between two vectors; Gram-Schmidt orthogonalization process; QR factorization; orthogonal transformation; orthogonal matrix; Least-squares approximation; normal equation.
Week 4	5.4: Least Squares and Data Fitting 5.5: Inner Product Spaces 6.1: Introduction to Determinants 6.2: Properties of the Determinant 6.3: Geometrical Interpretations of the Determinant; Cramer's Rule 7.1: Dynamical Systems and Eigenvectors: An Introductory Example	Inner products and norms in a general inner product space; Fourier analysis; Determinant of a (square) matrix; multilinearity; minors, cofactors, and adjoints; k-volumes; determinant as an expansion factor; Cramer's Rule; Discrete (linear) dynamical system; iteration of a matrix; eigenvectors and eigenvalues.
Week 5	7.2: Finding the Eigenvalues of a Matrix 7.3: Finding the Eigenvectors of a Matrix 7.4: Diagonalization 7.5: Complex Eigenvalues 7.6: Stability **Second hour exam** (Fri, July 30)	Eigenvalues and eigenvectors of a (square) matrix; characteristic polynomial; algebraic and geometric multiplicities; similarity of matrices; Diagonalization and the existence of a basis of eigenvectors; powers of a matrix; eigenvalues of a linear transformation; complex numbers, polar form and related algebra; rotation-dilation matrices revisited; trace and determinant; stability of a discrete linear dynamical system; phase portraits.
Week 6	8.1: Symmetric matrices 8.2: Quadratic Forms 9.1: An Introduction to Continuous Dynamical Systems 9.2: The Complex Case: Euler's Formula	Spectral Theorem; symmetric matrices and diagonalization by an orthonormal basis; quadratic forms; positive definiteness of a matrix; principal axes; applications to ellipses and hyperbolas; 2nd derivative test for functions of several variables in terms of eigenvalues; Systems of linear differential equations and their solutions.
Week 7	9.3: Linear Differential Operators and Linear Differential Equations 10: Topics in Partial Differential Equations	Eigenfunctions, characteristic polynomials; kernel and image of a linear differential operator; solutions to homogeneous and inhomogeneous linear differential equations; Partial differential equations – Laplace's equation, the heat equation, the wave equation.
Week 8	Review and **Final Exam** (Wed, Aug 18)	