Mathematics 223a Homework due October 21

October 14, 2008

1. Let G be a p-group. Consider the projection $G \to G^{ab} \otimes \mathbf{F}_p$ (alias: "G modulo the normal subgroup generated by commutators and p-th powers"). Let $x_1, x_2, \ldots, x_n \in G$ be a system of elements whose projection to $G^{ab} \otimes \mathbf{F}_p$ generates $G^{ab} \otimes \mathbf{F}_p$. Then $x_1, x_2, \ldots, x_n$ generates G. *Hint:* First do this when G is an abelian p-group. Next, let G be a finite group and $H \subset G$ a subgroup that is in the intersection of the center and the commutator subgroup of G (hence is normal in G) and assume you have a bunch of elements $x_1, x_2, \ldots, x_n \in G$ whose projection to G/H generates G/H; show that $x_1, x_2, \ldots, x_n$ generates G. Then conclude somehow.

2. Let G be a finite p-group. Show that G has a system of generators $x_1, x_2, \ldots, x_n \in G$ where $n = \dim_{\mathbf{F}_p} H^1(G, \mathbf{F}_p)$ (Note: we are thinking of $\mathbf{F}_p$ as G-module with trivial G-action).

3. Formulate and prove results analogous to items **(1)** and **(2)** above when G is a pro-p-group.