If H is an H-functor, I view inflations as injections. Call an element $\alpha \in H(*/K)$ *cyclic*, if $\alpha \in H(L/K)$ for some cyclic layer L/K. Notice that restriction preserves cyclicity

Let $H(L/K)$ be an H-functor satifying $|H(L/K)| \leq [L:K]$ and suppose that for each K there is a subgroup H_K of $H(*/K)$ and a homomorphism $\mathrm{inv}_K : H_K \to \mathbb{Q}/\mathbb{Z}$ satisfying the following properties:

(i) If n is the degree of some layer L/K, then H_K contains a cyclic element α of order n.

(ii) For every L/K we have $\mathrm{res}_L^K H_K \subset H_L$ and $\mathrm{inv}_L(\mathrm{res}_L^K(\alpha)) = [L:K]\,\mathrm{inv}_K(\alpha)$ for $\alpha \in H_K$

(iii) If $\alpha \in H_K$ is cyclic and $\mathrm{inv}_K(\alpha) = 0$, then $\alpha = 0$.

the map inv_K induces an isomorphism $H(L/K) \xrightarrow{\sim} \frac{1}{n}\mathbb{Z}/\mathbb{Z}$ for all cyclic layers L/K such that $H(L/K) \subset H_K$.

Then $H_K = H(*/K)$ and $\mathrm{inv}_K : H(*/K) \to \mathbb{Q}/\mathbb{Z}$ is injective for all K.

Proof: For given L/K, choose α as in (i). Then (ii) shows that $\mathrm{inv}_L(\mathrm{res}_L^K \alpha)) = 0$. By (iii) (applied to L instead of K) it follows that $\mathrm{res}_L^K(\alpha) = 0$, hence $\alpha \in H(L/K)$. Since α has order $n = [L:K]$ and we are assuming the 2d inequality, $H(L/K)$ is generated by α. Since $H(*/K) = \cup_L H(L/K)$, and our L was arbitrary, it follows that $H(*/K) \subset H_K$, every element of $H(*/K)$ is cyclic, and therefore (by (iii) applied to K) the kernel of inv_K is 0 as claimed.