

### Mathematics 22a Homework Problems 3

The following problems are due on **Monday, October 12**.

1-3. B & S, p. 74 #2.5, 2.7, 2.14.

4. Write down the matrix which represents the following linear transformations:

a)  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ;  $f((4, -3)) = (8, -6)$ ,  $f((0, 2)) = (0, 1)$ .

b)  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ;  $f((3, 1)) = (1, 3)$ ,  $f((2, 2)) = (2, 2)$ .

c)  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ;  $f((2, 1)) = (3, 5)$ ,  $f((-4, -2)) = (-6, -10)$ . (Having trouble with this one? Why?)

5. Let  $V$  be the vector space of degree 1 polynomials:

$$V = \{p(x) = ax + b \mid a, b \in \mathbb{R}\},$$

where addition and scalar multiplication are given by:

$$(p + q)(x) = p(x) + q(x); \quad (r \cdot p)(x) = rp(x).$$

Consider the linear transformation  $f : V \rightarrow V$  given by:

$$f(ax + b) = -bx + a.$$

For each of the following ordered pairs of vectors  $\beta = \{u, v\}$  in  $V$ , determine whether  $\beta$  is a basis for  $V$ . If it is a basis, then write down the matrix  $[f]_{\beta}$  representing  $f$  in the basis  $\beta$ . In each case, compute the trace and determinant of the matrix you get.

a)  $\beta = \{3x, 2x\}$

b)  $\beta = \{-1, \frac{1}{2}x\}$

c)  $\beta = \{x + 1, -3\}$

d)  $\beta = \{2x + 1, x + \frac{1}{2}\}$

6. If a linear transformation is represented by the matrix  $\begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix}$  with respect to some basis  $v_1, v_2$  of a certain two-dimensional vector space, what is the matrix representing it with respect to the basis  $u_1, u_2$  of the same vector space, if  $u_1, u_2$  are known to satisfy the equations  $v_1 = 2u_1 - u_2$  and  $v_2 = u_2 + 2u_1$ ?

7. Consider  $\beta_1 := \{(2, 2), (4, -1)\}$ , and  $\beta_2 := \{(2, 0), (0, -1)\}$ , two bases of  $\mathbb{R}^2$ .

a) Find the matrix  $B$  that expresses  $\beta_2$ -coordinates in terms of  $\beta_1$ -coordinates.

b) Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear map such that  $[f]_{\beta_2} := \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix}$ . Find the matrix  $[f]_{\beta_1}$ .

7. What is the characteristic polynomial of:

- A nilpotent matrix?
- A projection?
- A diagonal matrix?
- A rotation?