1.Let $f(x,y) = x^3 + 3x^2 - 12x + y^2 - 6y + 12$. For any $c \in \mathbb{R}$ we denote by $C_c \subset R^2$ the level curve $C_c = \{(x,y) \in \mathbb{R}^2 | f(x,y) = c\}$.

a)Show that the point $(2,2)$ is a smooth point of the curve C_0.

b)Show there exists $\epsilon > 0$ and differentiable functions $f(x), g(y)$ defined on the interval $(2-\epsilon, 2+\epsilon)$ such that the curve C_0 is equal to the graph of the function $y = f(x)$ and to also the graph of the function $x = g(y)$ in a sufficiently small open neighborhood of the point $(2,2)$.

c)Find the tangent line and the tangent space to C_0 at $(2,2)$

d)Find all $c \in \mathbb{R}$ such that the level curve $f(x,y) = c$ is not smooth.

e)For each $c \in \mathbb{R}$ such that the level curve $f(x,y) = c$ is not smooth find all the singular [not smooth] points on C_c

f) Let $p(x), q(y)$ be polinomial of degree m and n correspondingly. For any $c \in \mathbb{R}$ consider the curve $C_c = \{(x,y) \in \mathbb{R}^2 | p(x) - q(y)) = c\}$. Prove that there not more then mn values of c for which the curve C_c is not smooth.

2. a)Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Find for which $a, b \in \mathbb{R}$ there exists an open neighborhood $U \in M_2$ of the matrix A^2 and a continuously differentiable map $g : U \to M_2$ such that $g(X)^2 = X \forall X \in U$.

b)EXTRA CREDIT. Let $M_2(\mathbb{C})$ be the space of complex 2×2 matricies. $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Find for which $a, b \in \mathbb{C}$ there exists an open neighborhood $U \in M_2$ of the matrix A^5 and a continuously differentiable map $g : U \to M_2(\mathbb{C})$ such that $g(X)^5 = X \forall X \in U$.