MATH 23A 2-D MIDTERM EXAM,NOV.16

1. a) Let G be a subset of $\mathbb{R}^n$. Say what it means for G to be closed in terms of convergent sequences.

b) Show that G is closed if and only if $G^c = \mathbb{R}^n \setminus G$ is open.

c) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function. For any subset $V \subset \mathbb{R}$ we define the preimage $f^{-1}(V) \subset \mathbb{R}^n$ by $f^{-1}(V) = \{x \in \mathbb{R}^n | f(x) \in V\}$. Prove that f is continuous if and only if for any open subset V of $\mathbb{R}$ the preimage $F^{-1}(V)$ is an open subset of $\mathbb{R}^n$. (note: by part (b) this is equivalent to showing that $f^{-1}(G)$ is closed for any closed set G if and only if f is continuous so you can show that instead if you so desire)

2. Let $f : \mathbb{R} \to \mathbb{R}^2$ be defined by $f(t) = (e^t, e^{-t})$ and $g : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

$$g(x_1, x_2) = (\frac{x+y}{2}, \frac{x-y}{2}, xy).$$

Find $(g \circ f)'$ at $t = 0$ first by composing the functions and evaluating the derivative and then by the chain rule, verifying that the chain rule gives the same answer.

3.Let M_n be the vector space of $n \times n$ matricies and $F : M_n \to M_n$ be the map given by $F(A) = AA^T$ where A^T is the traspose of A. a)Prove that F is contineous.

b)Prove that F is differentiable and find $D(F)(A)$ for $A \in M_n$.

4.Let $\mathcal{B}_{an}(\mathbb{R}^n)$ be the space of antisymmetric bilinear forms on $\mathbb{R}^n$. Find the dimension of $\mathcal{B}_{an}(\mathbb{R}^n)$ for

a)n=1

b)n=2

c)n=3

d)n=10

5.Let $U \subset \mathbb{R}^n$ be an open set ,$F : U \to \mathbb{R}^n$ a continuosly differentiable function such that for any $u \in U$ the linear map $D(F)(u) : \mathbb{R}^n \to \mathbb{R}^n$ is an isomorphism. Fix $v \in \mathbb{R}^n$ and consider the function f on U given by $f(u) = |f(u) - v|^2$.

a)Show that the function f is differentiable

b)Prove that $D(f)(u) = 0$ if and only if $f(u) = v$