HOMEWORK FOR SEP. 29-TH

1. Let $f : B \to C, g : A \to B$ be maps such that the inverse maps $f^{-1} : C \to B, g^{-1} : B \to A$ exist. Prove that the map $f \circ g : A \to C$ is also invertible and show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

If $f : A \to B$ is a map we say that f has a right inverse if there exists $g : B \to A$ such that $f \circ g = Id_B$. We say that f has a left inverse if there exists a map $g : B \to A$ such that $g \circ f = Id_A$.

2. a) Show that a map $f : A \to B$ is is surjective iff f has a right inverse. Here iff := if and only if.

b) Show that a map $f : A \to B$ is is injective iff f has a left inverse.

c) Let $f : A \to B$ be a map which has both a right inverse $g' : B \to A$ and a left inverse $g'' : B \to A$. Show that f bijective and $g' = g''$.

Let S_n be the group of permutations of the set $[1, ..., n]$. For any permutation $\sigma \in S_n$ we denote by $l(\sigma)$ the number of pairs $1 \leq i < j \leq n$ such that $\sigma(i) > \sigma(j)$.

3. Show that for any permutation $\sigma \in S_n$ and any elementary transposition $s_k, 1 \leq k \leq n$ we have either

$l(s_k \circ \sigma) = l(\sigma) + 1$ or

$l(s_k \circ \sigma) = l(\sigma) - 1$.

For any permutation $\sigma \in S_n$ we define

$\epsilon(\sigma) := (-1)^{l}(\sigma)$.

4. Prove that for any permutations $\sigma', \sigma'' \in S_n$ we have

$\epsilon(\sigma) = \epsilon(\sigma')\epsilon(\sigma'')$ were

$\sigma := \sigma' \circ \sigma''$.