MATH 23A 1ST MIDTERM EXAM, OCT 19

Good luck!

1. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Find all 3×3 matrices such that $AB = BA$.
2. Let $V = \mathbb{R}^3, \{e_1, e_2, e_3\}$ be the standard basis of $V, v_1 = e_1 + e_2, v_2 = e_2 + e_3, v_3 = e_3$. Let $\{e^1, e^2, e^3\}, \{v^1, v^2, v^3\} \in V^\vee = V^*$ be the dual bases. Write v^2 as a linear combination of e^1, e^2, e^3.
3. Let $A = \left[\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right]$. Find A^7.
4. Let A, B be two 2×2 matricies such that $AB = 0$. Does this equality imply that $BA = 0$?
5. (a) Let V be a vector space of dimension d. Prove that one can not find a strictly increasing sequence of subspaces $L_1 \subsetneq L_2 \subsetneq \cdots \subsetneq L_r$ for $r > d + 1$.
   (b) Let A be a 2×2 matrix such that $A^5 = 0$. Prove that $A^2 = 0$.