PROOF TECHNIQUES, WITH EXAMPLES

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Induction: In induction, you prove the result for a small case (say, $n = 0$ or $n = 1$), and then show that if the result holds for n, then it holds for $n + 1$.

For instance, prove that

$$\sum_{i=1}^{n} i = \binom{n+1}{2} = \frac{n(n+1)}{2}. \tag{1}$$

Here's a famous example, the Fibonacci numbers. Define $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Prove that

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n. \tag{2}$$

Contraposition:

Contradiction: In a proof by contradiction, you assume that the statement you are trying to prove is false, and try to derive a contradiction.

Fact 1: If Bob has a bad hair day, he wears a hat.
Fact 2: If Bob wears a hat, he must be bald.
Fact 3: If Bob is bald, he has a good hair day.
Prove: Bob does not have a bad hair day.

Some typical situations where a proof by contradiction might work:

1. Prove that there is no solution for [something].
2. Prove that there does not exist [something(s)] such that [some statement(s)] is/are true.
3. Prove that there exists a unique [something] such that [some statement(s)] is/are true.
4. Prove that [something] is the maximum (or minimum) value of [a variable] such that [some statement(s) depending on the variable] is/are true.
5. Prove that a given function is one-to-one.

Prove that there is no solution to the following system of equations: $\begin{cases} x + y = 1 \\ 2x + 2y = 3 \end{cases}$

Example:

Counterexample:

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Date: October 2, 1999.