Homework problems for November 3-d

All the references are to the Chapter 3.

The following result was not yet proven in class, but we will prove it later. You are allowed to use it for the first homework problem.

Theorem. Any two norms $\|x\|', \|x\|'', x \in \mathbb{R}^n$ on $\mathbb{R}^n$ are equivalent.

1. a) Let $\|x\|$ be a norm on $\mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}$ and f be a linear function. Then f is continuous.

A hint. Use the example 5 from section 3.

b)Let U be a finite-dimensional vector space. Show that any two norms $\|v\|', \|v\|''$ on V are equivalent.

c) Let $(V, \|v\|_V), (W, \|w\|_W)$ be finite-dimensional vector spaces and $T : V \to W$ be a linear map. Then T there exists a constnt [=a positive real number] c such that $\|T(v)\|_W \leq c\|v\|_V \forall v \in V$.

A hint. Apply the part b) to the case when $U \subset W$ is the image of $T, \|u\|'$ is the restriction of $\|w\|_W$ to U and $\|x\|''$ is the norm on U defined by $\|u\|'' := min_{v \in V | T(v)=u} \|v\|_V$. [Of course you have to prove that $\|u\|', \|u\|''$ are norms on U].

2. Let M_n be the vector space of $n \times n$ matricies and $F : M_n \to M_n$ be the map given by $F(X) := X^2$.

a)Prove that F is continuous.

b)Prove that F is differentiable at X for any $X \in M_n$ and find the derivative $D(F)(X)$.

c)Find partial derivatives of the function F.

The rest of the problems are from Chapter 3.

3. Section 7 Numbers 2,5,6,.