Homework problems for December 6-th

If $F : \mathbb{R}^3 \to \mathbb{R}$ is a twice differentiable function we define the function $\Delta(f)$ on $\mathbb{R}^3$ by

$$\Delta(f) := \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

1)Let $r(x, y, z) := \sqrt{x^2 + y^2 + z^2}$, $u : \mathbb{R} \to \mathbb{R}$ be a twice differentiable even function and $f := u(r) : \mathbb{R}^3 \to \mathbb{R}$.

a)Show that f is a twice differential function .

b)Show that there exists a function $F : \mathbb{R} \to \mathbb{R}$ such that $\Delta(f) = F(r)$ and find the function F.

2)Let $U = \{(x, y) \in \mathbb{R}^2 | y \neq 0\}$ and f be a function on *U* given by $f(x, y) := x/y$. Find the Taylor polynomial $P_f^N(1, 1)(x, y)$.

3) Let f be a function on $\mathbb{R}^2$ given by

$$f(x, y) = \int_0^1 (1 + x^2)^{t^2 y} dt$$

. Find the Taylor polynomial $P_f^2(0, 0)(x, y)$

4)Let $f, g : \mathbb{R} \to \mathbb{R}$ be 3-times differentiable functions and $u(x, y) := f(x + g(y))$. Show that

$$\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial^2 x}$$

The rest of problems are from your book, Chapter 6.

Section 3 number 9, section 6 numbers 3,7,8 and section 8 number 1