MATH 23a, FALL 2002
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Homework Assignment # 1 (Final Version)
Due: September 27, 2002

Beginning with this problem set, we ask that you turn in four separate sheets (or sets of sheets) labelled A through D, so that the CA's may grade them in parallel. The individual problems are each labelled with one of A through D below.

1. Reading: Section 1.2 from Curtis and Chapter 1 and Section 2.1 from Fitzpatrick.

2. (A) Use mathematical induction to show that $\sum_{i=1}^{k}(2i-1) = k^2$.

3. (A) Verify that multiplication is well-defined for integers, as defined by equivalence classes of ordered pairs of natural numbers.

4. (B) Considering the integers as defined by equivalence classes of pairs of natural numbers, prove the existence of additive inverses. *(Hint: To do this, you will need first to name the additive identity correctly.)*

5. (C) Show that the field $\mathbb{Z}/2\mathbb{Z}$ is not an ordered field. That is, show that there is no possible choice for a set P of positive elements such that axioms P1–P3 hold.

6. (C) Show that additive inverses in fields are unique. That is, show that given $a \in F$, there exists a *unique* element $b \in F$ such that $a + b = 0$.

7. (D) Let $x > 0$ be a real number. Show that there is an integer k such that x may be represented in the form

   $$x = \sum_{i=k}^{\infty} a_i \cdot 10^{-i} = a_k \cdot 10^{-k} + a_{k+1} \cdot 10^{-k-1} + \cdots + a_{-1} \cdot 10^1 + a_0 \cdot 10^0 + a_1 \cdot 10^{-1} + a_2 \cdot 10^{-2} + \cdots$$

   where $a_i \in \{0, 1, 2, \ldots, 9\}$ for every $i \geq k$. Show that this representation is unique, except in the case where there exists some $n \in \mathbb{N}$ such that $10^n \cdot x \in \mathbb{N}$.

   (Hint: Use the Well-Ordering Principle and perhaps the Division Algorithm.)