MATH 23b, SPRING 2002
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Homework Assignment # 10
Due: May 3, 2002

Homework Assignment #10 (Version 2)

1. Read Edwards Sections 5.1–5.2, especially pp. 295–301.

2. The Gamma function.

   (a) Read Example 9 from Edwards, Section 4.6, defining the *Gamma function* as follows. For $x > 0$, define:

   $$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

   (b) Adapted from Edwards' problem 6.1, p. 282: Show that $\Gamma(1) = 1$. Use induction to prove that $\Gamma(n+1) = n!$, for all $n \in \mathbb{N}$, by showing that $\Gamma(x+1) = x\Gamma(x)$, for any $x > 0$.

   (c) Adapted from Edwards' problem 6.2, p. 282: Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. Use induction to show that this generalizes to:

   $$\Gamma(n + \frac{1}{2}) = \frac{1 \cdot 3 \cdot \cdots \cdot (2n-1)}{2^n}\sqrt{\pi}.$$

3. Follow the hints in Edwards' problem 6.15, p. 284 to show that the function $f(x) = \frac{\sin x}{x}$ is not absolutely integrable on $(0, \infty)$.

4. Adapted from Edwards' problem 1.13, p. 302:

   Let $F : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be given by $F(x, y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$.

   Let C_0 be the unit circle in $\mathbb{R}^2$ parametrized by the function $\gamma_0 : [0, 1] \longrightarrow \mathbb{R}^2$, where $\gamma_0(t) = (\cos 2\pi t, \sin 2\pi t)$.

   Let C_1 be the circle of radius 1 centered at $(1, 1)$ in $\mathbb{R}^2$ parametrized by the function $\gamma_1 : [0, 1] \longrightarrow \mathbb{R}^2$, where $\gamma_1(t) = (1 + \cos 2\pi t, 1 + \sin 2\pi t)$.

   (a) Show that $\int_{C_0} F = 2\pi$.

   (b) Show that $\int_{C_1} F = 0$.

5. Let $F(x, y, z) = (z^3 + 2xy, x^2, 3xz^2)$ be a vector field on $\mathbb{R}^3$. Show that F is conservative by computing the partial derivatives of its component functions, and find $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ such that $F = \nabla f$.

6. Evaluate $\int_C 2xyz\, dx + x^2 z\, dy + x^2 y\, dz$, where C is a piece-wise smooth oriented curve from $(1, 1, 1)$ to $(1, 2, 4)$.