

MATH 23a, FALL 2002  
THEORETICAL LINEAR ALGEBRA  
AND MULTIVARIABLE CALCULUS  
(Final Version) Homework Assignment #4  
Due: October 18, 2002

1. Read Sections 6, 8–9 (Chapter 2), 11–13 (from Chapter 3), and, looking ahead, 26 and 28 (from Chapter 8) of Curtis.

For problems 2 and 3, we consider the field  $F = \mathbb{Z}/7\mathbb{Z}$ . Recall that we may think of the elements of  $F$  as equivalence classes of integers, where  $a \sim b$  if and only if  $a - b$  is evenly divisible by 7, and where addition and multiplication are derived from  $\mathbb{Z}$ . (For example,  $[4] \cdot [5] = [6]$  because  $4 \cdot 5 = 20$ , which has remainder 6 when divided by 7.) For simplicity, we often drop the equivalence class notation and denote this field by  $\mathbb{Z}/7\mathbb{Z} = \{0, 1, 2, 3, 4, 5, 6\}$ .

2. (A) Let  $\mathbf{u} = (1, 0, 6)$ ,  $\mathbf{v} = (1, 2, 1)$ , and  $\mathbf{w} = (2, 1, 3)$  be three vectors in  $F^3$ , that is, the set of ordered triples with coordinates in  $F$ . Find coefficients  $a, b, c \in F$  to express the vector  $\mathbf{x} = (1, 2, 3)$  as a linear combination  $\mathbf{x} = a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$ .

*(Hint: Read Section 6 from Chapter 2.)*

3. (B) Find the kernel of the linear map  $L : (\mathbb{Z}/7\mathbb{Z})^3 \rightarrow (\mathbb{Z}/7\mathbb{Z})^3$  given by  $L(x, y, z) = (x + 2z, 2x + 3y + 4z, 4x + 3y + z)$ .
4. (C) Let  $P_n = \{p(x) = a_0 + a_1x + \cdots + a_nx^n \mid a_i \in \mathbb{R}, \forall i\}$  be the vector space of all polynomials of degree less than or equal to  $n$ . Consider the map  $L : P_n \rightarrow \mathbb{R}$  defined by  $L(p) = \int_0^1 p(x) dx$ .

(a) Show that  $L$  is a linear map.

(b) Determine  $\text{Im}(L)$ , and find a basis.

5. (D) Show that if  $W$  is a subspace of  $V$  and  $\dim(V) < \infty$ , then  $\dim(W) \leq \dim(V)$ . (Part of this problem is showing that  $W$  has a basis. Do this constructively by choosing vectors successively.)