Math 23b, 2003.
Solution Set 1, Question 2.

CA: Joshua Reyes

Question 2. Let $A \subset \mathbb{R}^n$ be compact, and suppose that $B \subset A$ is closed. Use the "open cover" definition to show that B is compact.

Solution. So, take an open cover of B; call it $U = \{U_n\}$ for kicks. Just for good measure, let's extend it to make sure it covers A, too, by throwing in B^c. This is legit since the complement of a closed set is open. More than B plus its complement'll hit everything, including A. But since A is compact, we can hack away until we find a finite subcover $U^* \subset U \cup B^c$. And of course, $B \subset A \subset U^*$. But Since B^c doesn't do anything for B we can throw that out, too, leaving $U^* - B^c$. And finite minus something is even *more* finite. We've got ourselves a finite open cover of B. So, who wouldn't conclude that B is compact, too?