

**Math 23a, 2002.**

**Solution Set 3, Question 5.**

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**Question 5.** Suppose  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is differentiable and has an inverse function  $f^{-1} : \mathbb{R}^n \rightarrow \mathbb{R}^n$  which is also differentiable. Show that:

$$[J(f^{-1})](a) = [(Jf)(f^{-1}(a))]^{-1}.$$

**Answer.** Their composition  $h = f \circ f^{-1}$  is much easier to work with since it's the linear function that sends  $a \mapsto I(a) = a$  (better known as the identity). Many of you showed or recalled from lecture or made it up and said that  $Jh = JI = I$  which is totally true for every conceivable  $a$ . However, if we try that the other way round, that nifty li'l chain rule tells us

$$I = Jh(a) = (Jf)(f^{-1}(a)) \cdot (Jf^{-1})(a).$$

We're in luck because both functions are differentiable. And since they're square matrices that multiply to the identity, they're inverses. Some of you pointed out that since  $\det (Jf)(f^{-1}(a)) \det (Jf^{-1})(a) = \det(I) = 1$  neither determinant can be identically zero and therefore both matrices are invertible. So right multiply thru' by  $[J(f^{-1})(a)]^{-1}$  and whoa! we've reached our conclusion.

Just as an aside to those out there who went down the road to the (decidedly unhappy) world of limits definitions, that's completely cool (if not harder to read), too. But you should remember that you can't just rip things out a limit without saying why. So please try write out your steps in words, not just symbols. It'll be easier to study from your sets later, too (and make life easier for whoever's grading your problem sets/midterms; and a happy grader is a nicer grader, right?). And oh! never, never, never pull things that depend on a limit to the other side of an equation without taking the limit operator with you.