

MATH 23b, SPRING 2003  
THEORETICAL LINEAR ALGEBRA  
AND MULTIVARIABLE CALCULUS  
Homework Assignment # 5  
Due: March 7, 2003

Homework Assignment #5 (Final Version)

1. Read Fitzpatrick, Chapters 16 and 17.

2. (A) (previously known as #4.4)

Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by  $f(x, y) = (\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2})$ . Show that  $f$  is locally invertible in a neighborhood of every point except the origin, and compute  $f^{-1}$  explicitly.

3. (B) A variation on problem # 16.3.3 from Fitzpatrick:

Suppose  $\varphi : \mathbb{R}^3 \rightarrow \mathbb{R}$  and  $\psi : \mathbb{R}^3 \rightarrow \mathbb{R}$  are continuously differentiable. Define  $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by:

$$F(\mathbf{x}) = (\varphi(\mathbf{x}), \psi(\mathbf{x}), 1 + \varphi(\mathbf{x})\psi(\mathbf{x}) + \varphi(\mathbf{x})^3).$$

(a) Explain analytically (using the Jacobian) why there is no point  $\mathbf{x} \in \mathbb{R}^3$  at which the assumptions of the Inverse Function Theorem hold for  $F$ .

(b) Explain geometrically (using the topology of Euclidean space) why there is no point  $\mathbf{x} \in \mathbb{R}^3$  at which the conclusion of the Inverse Function Theorem holds for  $F$ .

4. (C) Show that the equations  $\sin(x+z) + \log yz^2 = 0$  and  $e^{x+z} + yz = 0$  define  $z$  implicitly as a function of  $x$  and  $y$  near the point  $(1, 1, -1)$ .

5. (D) Consider the set  $S$  of points in  $\mathbb{R}^5$  defined by the two equations:

$$xu^2 + yzv + x^2z = 3$$

$$xyv^3 + 2zu - u^2v^2 = 2$$

Show that there is a neighborhood of the point  $(1, 1, 1, 1, 1) \in S$  and a differentiable function  $h : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  such that  $h(x, y, z) = (u, v)$  on the neighborhood, and find  $Jh(1, 1, 1)$ .