Solution for HW1, part A

Geoff Anderson
granders@fas.harvard.edu

Problem 2. (A) asks you to prove by induction that, for all positive integers k,

$$\sum_{i=1}^{k}(2i-1) = k^2$$

I'll give a few examples of possible proofs, since style of exposition was more of a problem than the solution itself.

Solution 1

Claim: For any $k \in \mathbb{N}$,

$$\sum_{i=1}^{k}(2i-1).$$

Proof: By induction on k. Let $F(k) =^{\text{def}} \sum_{i=1}^{k}(2i-1)$. In order to proceed with the induction, we need to check $F(1) = 1$ and that $F(k) = k^2 \Rightarrow F(k+1) = (k+1)^2$.

Base case: $F(1) = 2 - 1 = 1$

Inductive step: Suppose $F(k) = k^2$ (this is our *inductive hypothesis*). Then

$$F(k+1) = F(k) + (2k+1) = k^2 + (2k+1) = (k+1)^2$$

This completes the induction, so our claim follows.

Solution 2

Claim: Let $P = \{k \in \mathbb{N} \mid \sum_{i=1}^{k}(2i-1) = k^2\}$. Then $P = \mathbb{N}$

Proof: By induction. In order to do the induction, we need to show $1 \in P$ and that $k \in P \Rightarrow (k+1) \in P$. Having shown this, Peano's fifth axiom will insure $P = \mathbb{N}$.

...the rest is basically the same as in the first proof ...

Solution 3

Claim: Let $P(k)$ be the statement "$\sum_{i=1}^{k}(2i-1) = k^2$". Then $P(k)$ holds true for all positive integers, k.

Proof: We must show truth $P(1)$ is true and that truth of $P(k) \Rightarrow$ truth of $P(k+1)$.

...etc ...

Some observations

- Note that in solution 1, I defined $F(k)$ so that the reader would know what I was talking about when I later referred to $F(k)$. Most of you did *not* bother to define your notation. Make sure to define any variables, sets etc before you start using them willy-nilly in your proof. E.g., "let P be the set of positive integers", or, "let $P = \{k \in \mathbb{N} | k = k + 1\}$".

- A somewhat common mistake occured in the conclusion. It usually read something like, "we've shown $F(1) = 1$ and $F(k+1) = (k+1)^2$ for all $k \in \mathbb{N}$, so it follows by induction...". NO! We've actually shown that $F(1) = 1$ and that $F(k) = k^2 \Rightarrow F(k+1) = (k+1)^2 \forall k \in \mathbb{N}$. This is the essence of induction, so if you made this mistake, ponder the distinction between these statements sometime while you're in the shower or falling asleep or something.