Math 23a Solution Set #2, Part D

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Problem 6

(a) *Recall the vector space $C[0,1] = \{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$. Now, for each $c \in \mathbb{R}$, consider the set of vectors defined by*

$$V(c) = \{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous, and } f(0) = c\}.$$

For which values of c is $V(c)$ a subspace of $C[0,1]$? Explain.

Solution: $V(c)$ is a subspace of $C[0,1]$ if and only if $c = 0$.

First we show that $V(0)$ is a subspace of $C[0,1]$. It is closed under addition because $(f+g)(0) = f(0) + g(0) = 0 + 0 = 0$ for all $f, g \in V(0)$; it is closed under scalar multiplication because $(cf)(0) = cf(0) = c \cdot 0 = 0$ for all $c \in \mathbb{R}, f \in V(0)$. Thus $V(0)$ is a subspace of $C[0,1]$.

If $c \neq 0$, then $V(c)$ is not a subspace. To see this, suppose that $V(c)$ is a subspace for some c. Then for any $f \in V(c)$, we have $f + f \in V(c)$, hence $(f+f)(0) = f(0) + f(0) = c + c = c$, which occurs if and only if $c = 0$.

Remarks

- Everyone showed that $V(c)$ *is not* a subspace of $C[0,1]$ when $c \neq 0$. But few people actually showed that $V(c)$ *is* a subspace when $c = 0$. I'm sure all of you could have done this; it's just a matter of showing that $V(0)$ is closed under addition and scalar multiplication.