Math 23a, Fall 2003

Problem Set 5, Part D
Solutions written by Tseno Tselkov

Problem 8: *Let $P_n(\mathbb{R})$ be the vector space of polynomials (with real coefficients) of degree less than or equal to n, and let $n \geq 3$. Find a basis for $P_n(\mathbb{R})/P_2(\mathbb{R})$.*

Proof. Let $U = P_n(\mathbb{R})$ and $V = P_2(\mathbb{R})$. Then directly from the second proof of Part E of this problem set we get that the cosets $x^3 + V, x^4 + V, \dots, x^n + V$ form a basis for U/V.

Alternatively, we can check this directly. Firstly, these are linearly independent as $\lambda_3(x^3 + V) + \ldots + \lambda_n(x^n + V) = \lambda_3 x^3 + \ldots + \lambda_n x^n + V \neq 0 + U$ since $\lambda_3 x^3 + \ldots + \lambda_n x^n \notin V = P_2(\mathbb{R})$ unless $\lambda_3 = \ldots = \lambda_n = 0$. Secondly, these cosets span the quotient space as for any $f = a_n x^n + \ldots a_1 x + a_0 \in P_n(\mathbb{R})$ we clearly have $f \in a_3(x^3 + V) + \ldots + a_n(x^n + V)$. $\square$