Solution for HW6, part A
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Problem 2

Suppose λ is an eigenvalue for the linear transformation $A : V \to V$.

(a) Show that λ^2 is an eigenvalue for A^2.

(b) If A is invertible, show that λ^{-1} is an eigenvalue for A^{-1}. (What if $\lambda = 0$?)

Solution

(a) Since λ is an eigenvalue, there exists some $v \in V$ such that

$$Av = \lambda \cdot v.$$

Now, $A^2 v = \lambda^2 \cdot v$, so λ^2 is an eigenvalue of A^2.

(b) Noting that $A^{-1}Av = v$, we see $v = A^{-1}(\lambda \cdot v) = \lambda \cdot A^{-1}v$. Hence, $A^{-1}v = \lambda^{-1} \cdot v$, and λ^{-1} is an eigenvalue of A^{-1}.

If $\lambda = 0$, then A has a nontrivial kernel, implying that A is not invertible.