MATH 23a, FALL 2003
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Final Exam: True/False Solutions
January 24, 2004

1. True or False (44 points, 2 each)

True	There exist non-trivial vector spaces with only finitely many vectors.
True	If $L : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation and $n > m$, then $Ker(L)$ is non-trivial.
True	If $A : U \longrightarrow V$ and $B : V \longrightarrow W$ are both injective linear maps, then $B \circ A : U \longrightarrow W$ is also injective.
True	If U and V are subspaces of a vector space, then $(U + V)/V \cong U/(U \cap V)$.
False	Every skew-symmetric multilinear form $f : V^n \longrightarrow F$ is alternating.
True	If $dim(V) = m$ and $f : V^n \longrightarrow F$ is an alternating form with $n > m$, then $f = 0$.
True	If $A \in O_n(\mathbb{R})$, then $det(A) = \pm 1$.
True	Every Cauchy sequence of integers converges to an integer.
True	Every bounded set of integers has a least element.
False	Every bounded set of rational numbers has a least element.
False	Every bounded set of real numbers has a least element.
True	If $A, B \in M_n(\mathbb{R})$, then $det(AB) = det(BA)$.
True	If $A, B \in M_n(\mathbb{R})$ are similar, then $det(A) = det(B)$.
True	If $A, B \in M_n(\mathbb{R})$ are similar, then $\text{Spec}(A) = \text{Spec}(B)$.
True	If $\sigma = (123)(456) \in S_7$, then $\text{sgn}(\sigma) = +1$.
True	If $\sigma = (123)(456) \in S_7$, then σ is bijective as a function from the set $X = \{1, 2, 3, 4, 5, 6, 7\}$ to itself.
False	If $\sigma = (123)(456) \in S_7$, then $\sigma = \sigma^{-1}$.

True	Given a linear transformation $A : V \to V$, any set of non-zero eigenvectors for A with distinct eigenvalues is linearly independent.
True	Given an inner-product space V, any set of mutually orthogonal non-zero vectors is linearly independent.
True	If V is a subspace of an inner product space U, then $V \cap V^{\perp} = \{\mathbf{0}\}$.
False	If V is an inner product space and $\mathbf{u}, \mathbf{v} \in V$, then $\|\|\mathbf{u} + \mathbf{v}\|\| = \|\|\mathbf{u}\|\| + \|\|\mathbf{v}\|\|$.
False	If V is an inner product space and $\mathbf{u}, \mathbf{v} \in V$ satisfy $\langle \mathbf{u}, \mathbf{v} \rangle = 0$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.