

# Review Problems

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Answer true or false.

1. If  $A^{-1}$  and  $B^{-1}$  exist and commute, then  $A$  and  $B$  commute.
2. If  $A, B$  are  $n \times n$  matrices,  $A \neq 0$ , and  $AB = 0$ , then  $B = 0$ .
3. If  $A$  and  $B$  commute, and  $B^{-1}$  exists, then  $A$  and  $B^{-1}$  commute.
4.  $(A^t)^{-1} = (A^{-1})^t$ .
5.  $(AB)^t = A^t B^t$ .
6.  $(AB)^{-1} = A^{-1} B^{-1}$ .
7.  $(A + B)^2 = A^2 + 2AB + B^2$ .
8. If  $W$  is a subspace of a finite-dimensional vector space  $V$  and  $\dim W = \dim V$ , then  $W = V$ .
9. If  $U, W$  are subspaces of  $V$  for which  $\dim(U+W) = \dim(U) + \dim(W)$ , then  $U \cap W = \{0\}$ .
10. If  $U, W$  are subspaces of  $V$ , which of the following are subspaces of  $V$ , and which are not:
  - $U \cap W$
  - $U + W$
  - $U \cup W$
11. Let  $V$  be the vector space of all upper-triangular matrices with entries in  $F$ . Then  $\dim V =$ 
  - $n^2$
  - $2n + 1$
  - $\frac{n(n-1)}{2}$
  - $\frac{n(n+1)}{2}$

12. If  $\tau_1, \tau_2, \dots, \tau_m$  are transpositions in  $S_n$ , then  $\text{sgn}(\tau_m \tau_{m-1} \dots \tau_1) = (-1)^{m-n}$ .
13. If  $A$  is  $n$  by  $n$ , and every product  $A_{1,\pi(1)} \dots A_{n,\pi(n)}$  is zero, then  $A$  has a zero row.
14. If  $\tau$  is a transposition, then  $\tau = \tau^{-1}$ .
15.  $\det(A^t) = \frac{1}{\det(A)}$ , for an invertible matrix  $A$ .
16. If  $A$  is a square matrix with integer entries, then  $\det A$  is an integer.
17. If  $\pi_1 = (2\ 1\ 4\ 3)$  and  $\pi_2 = (1\ 3\ 4\ 2)$ , find  $\pi_1 \pi_2$ .
18. If  $\sigma = (5\ 2\ 3\ 4\ 1)$ , then  $\text{sgn}(\sigma) = 1$ .
19. If  $A$  is  $n \times n$  and  $\dim(\text{Ker}(A)) = n$ , then  $A = 0$ .
20. If  $A, B$  are  $n \times n$  matrices, then  $\text{Ker}(BA) \subset \text{Ker}(A)$ .
21. There exists a linear transformation  $T : V \rightarrow V$  with representations  $A$  and  $B$  such that  $\det A \neq \det B$ .
22. Any matrix is similar to a diagonal matrix.
23. The determinant of a matrix is zero iff one of its eigenvalues is 0.
24. If  $A$  is a  $2 \times 2$  matrix, then  $\det(\lambda I - A) = \lambda^2 - \text{tr}A \cdot \lambda + \det A$ .
25.  $(x, y) = x^t \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} y$  is an inner-product.
26. Any orthonormal family is linearly independent.
27. Any orthogonal family is linearly independent.
28. If the family  $\{(1, 1), (1, -1)\}$  is orthonormalized, what does one obtain?
29. If  $u, v \in V$ , then  $\|u + v\|^2 = \|u\|^2 + \|v\|^2$ .
30. Every finite-dimensional inner-product space has an orthonormal basis.
31. Not every orthonormal list of vectors in  $V$  can be extended to an orthonormal basis of  $V$ .

32. Each operator on  $V$  has at most  $(\dim V - 1)$  distinct eigenvalues.
33. Eigenvectors corresponding to distinct eigenvalues are linearly independent.
34. Eigenvectors corresponding to distinct eigenvalues are orthogonal.
35. If  $\{v_1, \dots, v_n\}$  spans  $V$ , and  $T : V \rightarrow W$  is surjective, then  $\{T(v_1), \dots, T(v_n)\}$  spans  $W$ .
36. If  $\{v_1, \dots, v_n\}$  is linearly independent in  $V$ , and  $T : V \rightarrow W$  is surjective, then the set  $\{T(v_1), \dots, T(v_n)\}$  is linearly independent in  $W$ .
37. If  $V$  is finite dimensional, then any spanning list of vectors in  $V$  with length  $\dim V$  is a basis of  $V$ .
38. There exist  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  with no eigenvalues.
39. For any Cauchy sequence  $[\{a_n\}_1^\infty]$  with a finite number of 0s,  $\exists N \in \mathbb{N}$ ,  $L > 0$ ,  $L \in \mathbb{Q}$ , s.t.  $|a_n| > L \forall n > N$ .
40. If  $\{v_1, \dots, v_n\}$  is an orthonormal set in  $V$  and  $\dim(V) = n$ , then  $\{v_1, \dots, v_n\}$  is a basis for  $V$ .