

3C

Here is a simple formula for figuring out problems like these:

$$\# \text{ of distinct subspaces} = \frac{\text{number of possible bases to choose}}{\text{bases per subspace}} \quad (1)$$

The units work out here—the “per subspace” gets up in the numerator. So for part a, the fraction is

$$\# \text{ of distinct one dimensional subspaces} = \frac{p^n - 1}{p - 1} \quad (2)$$

where the -1 expresses the fact that you can't choose the zero vector as a basis vector.

And for part b, the fraction is

$$\# \text{ of distinct two - dimensional subspaces} = \frac{(p^n - 1)(p^n - p)}{(p^2 - 1)(p^2 - p)} \quad (3)$$

You multiply by $(p^n - p)$ in the numerator because there are p many vectors that are in the span of the first basis vector you chose.