

Math 23a Theoretical Linear Algebra and Multivariable Calculus I

MIDTERM EXAM 1 - PRACTICE EXAM

In the following problems you can use any of the results we have proved in class, if you state them clearly before using them.

Problem 1: (a) State the Principle of Mathematical Induction.

(b) Prove, by induction, that the following identity is true for every $n \geq 1$:

$$-1^2 + 2^2 - 3^2 + 4^2 - \dots + (-1)^n n^2 = \frac{1}{2}(-1)^n n(n+1).$$

Problem 2: (a) Define what is an *order* relation.

(b) Define what is an *ordered* field.

(c) Prove that $\mathbb{Z}/7\mathbb{Z}$ is not an ordered field.

Problem 3: Which of the following subsets $U \subset V$ is a subspace? (Justify your answer)

(a) $V = \mathbb{R}^3$, $U = \left\{ \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \in \mathbb{R}^3 \mid \alpha_1 = 1 \right\} \subset V.$

(b) $V = \mathbb{R}^3$, $U = \left\{ \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \in \mathbb{R}^3 \mid \alpha_1 + 2\alpha_2 = 0 \right\} \subset V.$

(c) $V = \mathbb{R}^3$, $U = \left\{ \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \in \mathbb{R}^3 \mid \alpha_1^2 = 2\alpha_2 \right\} \subset V.$

(d) V is the set of all functions $f : \mathbb{R} \rightarrow \mathbb{R}$, considered as a vector space over \mathbb{Q} , and $U = \{f \in \text{Fun}(\mathbb{R}) \mid f(1/2) \in \mathbb{Q}\} \subset V.$

Problem 4: Let V be a vector space such that $\dim V = n$.

(a) Let v_1, \dots, v_n be vectors of V such that $\text{span}(v_1, \dots, v_n) = V$. Prove that (v_1, \dots, v_n) is a basis of V .

(b) Let v_1, \dots, v_n be linearly independent vectors of V . Prove that (v_1, \dots, v_n) is a basis of V .

Problem 5: Which of the following lists of vectors of \mathbb{R}^2 is linearly independent and which is not? (Justify your answer)

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$