PROBLEM 2, PROBLEM SET 6

Part a)
Note that $I((x-1)^2) = \frac{-1}{3}(x-1)^3$, $I(x-1) = \frac{1}{2}(x-1)^2$ and $I(1) = x-1$. It follows that the corresponding matrix is

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}.$$

Part b)
Note that $1 = 1 + 0+)$, $x - 1 = -1 + x + 0 + 0$, $(x-1)^2 = -1 + 3x - 3x^2 + x^3$. It follows that the change of basis matrix is

$$P = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}.$$

Now note that $1 = 1+0+0+0$, $x-1 = -1+x+0+0$, $(x-1)^2 = 1-2x+x^2+0$, $(x-1)^3 = -1+3x-3x^2+x^3$. It follows that the change of basis matrix is

$$Q = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Matrix multiplication shows us that

$$B_I P = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & \frac{1}{2} & -1 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} = QA_I.$$

Hence, $Q^{-1}B_I P = A_I$, and we're done :-).