Problem Set 4, Problem 3 Solution

(a)The subset A_3 of polynomials of degree exactly 3 in $F_3[x]$ is not a subspace.

Counterexample:
Let $f(x) = x^3 + x^2$, $g(x) = -x^3$, so $f, g \in A_3$ but $(f + g)(x) = x^2 \notin A_3$, so A_3 is not closed under addition and is therefore not a subspace.

(b) The subset P_3 of polynomials of degree less than or equal to 3 is a subspace.

Let $f(x) = a_3x^3 + a_2x^2 + a_1x + a_3$ and $g(x) = b_3x^3 + b_2x^2 + b_1x + b_0$ with $f, g \in P_3$ and $a_j, b_j \in F$.

Closed under addition:

$(f + g)(x) = (a_3 + b_3)x^3 + (a_2 + b_2)x^2 + (a_1 + b_1)x + (a_0 + b_0) \in P_3$

Closed under scalar multiplication:

Let $c \in F$. $(cf)(x) = (ca_3)x^3 + (ca_2)x^2 + (ca_1)x + ca_0 \in P_3$

Non-empty:

$x^3 \in P_3$, so P_3 is non-empty.

Therefore P_3 is a subspace of $F[x]$.