

# Math 23. P-SET 6

## Question 1

(a) Consider  $P \in P_3$

$$I \circ D(P) = \int_1^x \frac{dP}{dy} dy = P(x) - P(1)$$

$$\text{So } I \circ D: P_3 \rightarrow P_3$$

Now take  $P \in P_2$

$$D \circ I(P) = \frac{d}{dx} \int_1^x P(y) dy = P(x)$$

$$\text{So } D \circ I: P_2 \rightarrow P_2$$

(b)  $B_2 = (1, x, x^2)$      $B_3 = (1, x, x^2, x^3)$

$$I(1) = x-1 \quad I(x) = \frac{x^2}{2} - \frac{1}{2} \quad I(x^2) = \frac{x^3}{3} - \frac{1}{3}$$

$$\text{So } A_I = \begin{bmatrix} -1 & -\frac{1}{2} & -\frac{1}{3} \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$D(1) = 0 \quad D(x) = 1 \quad D(x^2) = 2x \quad D(x^3) = 3x^2$$

$$\text{So } A_D = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$(C) \quad D \circ I = \mathbb{1} : \mathbb{P}_2 \rightarrow \mathbb{P}_2$$

$$\text{So } A_{D \circ I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I \circ D(1) = 0 \quad I \circ D(x) = x-1 \quad I \circ D(x^2) = x^2-1 \quad I \circ D(x^3) = x^3-1$$

$$\text{So } A_{I \circ D} = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(d) \quad A_I \cdot A_D = \begin{bmatrix} -1 & -\frac{1}{2} & -\frac{1}{3} \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_{I \circ D}$$

$$A_D \cdot A_I = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & -\frac{1}{2} & -\frac{1}{3} \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A_{D \circ I}$$