PROBLEM SET 7, PROBLEM 5

(a) <u>False</u>

Here is a counterexample:

Let $A=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $B=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$. So, $A+B=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\det A = 1$, $\det B = 1$, $\det(A+B)=0$

Therefore, $\det A + \det B = \det(A+B)$

(b) <u>True</u>

For this part, we use the property that $\det(PQ)=\det P \cdot \det Q$

$\det((A+B)^2) = \det((A+B)\cdot(A+B)) = \det(A+B)\det(A+B)$

$= [\det(A+B)]^2$

(c) <u>False</u>

In this part, be aware that $(A+B)^2 \neq A^2+2AB+B^2$

$(A+B)^2 = A^2+AB+BA+B^2$

AB may not necessarily equal BA.

Let $A=\begin{bmatrix}0&1\\0&0\end{bmatrix}$, $B=\begin{bmatrix}0&0\\1&0\end{bmatrix}$, So, $A+B=\begin{bmatrix}0&1\\1&0\end{bmatrix}$.

$(A+B)^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\det(A+B)^2 = 1$.

But $A^2=B^2=\begin{bmatrix}0&0\\0&0\end{bmatrix}$. $AB=\begin{bmatrix}1&0\\0&0\end{bmatrix}$ $\Rightarrow$ $A^2+2AB+B^2=\begin{bmatrix}2&0\\0&0\end{bmatrix}$

$\det(A^2+2AB+B^2)=0$.

(d) True

If $C = A+B$, $c_{ij} = a_{ij} + b_{ij} = b_{ij} + a_{ij}$. So, $C = B+A$.

Since $A+B = B+A$, $\det(A+B) = \det(B+A)$

(e) True

$\det(AB) = (\det A)(\det B) = (\det B)(\det A) = \det(BA)$

(f) True

If A is invertible, there exists a matrix A^{-1} s.t

$$\underbrace{A \cdot A^{-1} = \mathbb{1}}\ = A^{-1} \cdot A.$$

Taking the determinant of both sides yields

$$\det(A \cdot A^{-1}) = \det(\mathbb{1})$$

$$(\det A)(\det A^{-1}) = 1$$

$$\det A^{-1} = \frac{1}{\det A} = (\det A)^{-1}$$