

# P-SET 8 Problem 1

(a) The characteristic polynomial is

$$(4-\lambda)(-2-\lambda)+5 = (\lambda-3)(\lambda+1)$$

$$\Rightarrow \lambda_1 = 3 \quad \lambda_2 = -1$$

Plugging this into  $M\begin{pmatrix} a \\ b \end{pmatrix} = \lambda\begin{pmatrix} a \\ b \end{pmatrix}$

$$\text{we get } 4a+5b=3a \Rightarrow a=-5b$$

$$\text{so } v_1 = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$\text{and } 4a+5b=-a \Rightarrow a=-b$$

$$\text{so } v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(b) The char. poly. is  $(-3-\lambda)(1-\lambda)+4 = (\lambda+1)^2$

$$\text{so } \lambda_1 = -1$$

Plugging this in, we have

$$-3a-2b = -a \Rightarrow a = -b$$

$$\text{so } v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

This is our only eigenvalue & eigenvector.

(c) Char. poly. is  $(1-\lambda)(-2-\lambda)$

$$\Rightarrow \lambda_1 = 1 \quad \lambda_2 = -2$$

Plugging in for  $\lambda_1$  we have

$$a-2b=b \Rightarrow a=3b$$

$$\text{so } v_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

For  $\lambda_2$ ,

$$a-2b=-2b \Rightarrow a=0$$

$$\text{so } v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$