P-SET 9

Question 1

A) a) if $z = a+bi$ $a,b \in \mathbb{R}$, $\bar{z} = a-bi$

$z\bar{z} = (a+bi)(a-bi) = a^2+b^2 \in \mathbb{R}$

$z+\bar{z} = a+bi + a-bi = 2a \in \mathbb{R}$ $\frac{1}{i}(z-\bar{z}) = \frac{1}{i}(2bi) = 2b \in \mathbb{R}$

b) $w = c+di$ $\overline{zw} = \overline{(a+bi)(c+di)} = \overline{(ac-bd)+(ad+bc)i} = (ac-bd)-(ad+bc)i$

$\bar{z}\bar{w} = (a-bi)(c-di) = (ac-bd) - (ad+bc)i = \overline{zw}$

$\overline{z+w} = a+c-(b+d)i = a-bi+c-di = \bar{z}+\bar{w}$

B) a) Consider v_k the kth entry in V. $v_k \in \mathbb{C}$

$(v+\bar{v})_k = v_k + \bar{v}_k \in \mathbb{R}$ by A) a) This is true $\forall k \Rightarrow v+\bar{v} \in \mathbb{R}^n$

Likewise $\frac{1}{i}(v-\bar{v})_k = \frac{1}{i}(v_k - \bar{v}_k) \in \mathbb{R} \Rightarrow \frac{1}{i}(v-\bar{v}) \in \mathbb{R}^n$

(b) Take $(Av)_k$ to be the kth entry of this vector.

If $A = \begin{bmatrix} - r_1 - \\ \vdots \\ - r_n - \end{bmatrix}$ Then $\left(\overline{Av}\right)_k = \overline{r_k \cdot v} = \overline{z_{k1}v_1 + \cdots + z_{kn}v_n} = \bar{z}_{k1}\bar{v}_1 + \cdots + \bar{z}_{kn}\bar{v}_n$

By A)

But $(\bar{A}\bar{v})_k = \bar{z}_{k1}\bar{v}_1 + \cdots + \bar{z}_{kn}\bar{v}_n$

So $(\bar{A}\bar{v})_k = (\overline{Av})_k$ $\forall k$ $\Rightarrow \bar{A}\bar{v} = \overline{Av}$

$(\overline{v+w})_k = \overline{v_k + w_k} = \bar{v}_k + \bar{w}_k \Rightarrow \overline{v+w} = \bar{v}+\bar{w}$

$\blacksquare$