

## Mathematics 22a Homework Problems 9

The following problems are due on **Wednesday, December 2**. Please read B & S, pages 175 - 178. Skim 178- 183. Pages 191 - 195 are extremely sophisticated mathematics, so read only if you're feeling ambitious. The section on Kepler motion, pages 195-197, on the other hand, is readable and useful for physicists (though you will not be tested on it). Finally, you should try to read sections 5.5 and 5.6.

1. Problems 3 and 4 from HW 8.

2. Let  $f$  and  $g$  be functions given below. In each case, calculate the derivative  $d(g \circ f)_p$ .

a)  $f : \mathbf{R} \rightarrow \mathbf{R}^2$  given by the formula  $f(x) = (x, \sin(x))$  and  $g : \mathbf{R}^2 \rightarrow \mathbf{R}$  given by the formula  $g(x, y) = x^3 y^5$  at the point  $p = \pi$ .

b)  $f : \mathbf{R}^2 \rightarrow \mathbf{R}$  given by the formula  $f(x, y) = x^2 - y^2$  and  $g : \mathbf{R} \rightarrow \mathbf{R}^2$  given by the formula  $g(x) = (\sin(x), \cos(x))$  at the point  $p = (0, 0)$ .

c)  $f : \mathbf{R} \rightarrow \mathbf{R}^2$  given by the formula  $f(x) = (e^x, e^{-x})$  and  $g : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  given by the formula  $g(x, y) = ((x + y)/2, (x - y)/2)$  at the point  $p = 1$ .

d)  $f : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  given by the formula  $f(x, y) = (x^3 - xy^2, x^2y - y^3)$  and  $g : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  given by the formula  $g(x, y) = (y^5, -x^5)$  at the point  $p = (1, 1)$ .

4. Let  $f : V \rightarrow V$  be a function that is differentiable at the point  $p \in V$ . Suppose that  $f$  has an inverse function  $g : V \rightarrow V$  (so that  $g \circ f(x) = f \circ g(x) = x, \forall x \in V$ ) that is differentiable at the point  $f(p)$ . Using the chain rule, express the derivative  $dg_{f(p)}$  in terms of the derivative  $df_p$ .

6. B & S p. 215, # 5.5.

7. B & S p. 215, # 5.7.