1. Let $f : [x_0 - a, x_0 + a] \to \mathbb{R}$ be a continuously differentiable function and $C > 0$ be a number such that $f'(x) > C$ for all $x \in [x_0 - a, x_0 + a], y_0 := f(x_0)$. Show that for any $y \in \mathbb{R}, |y - y_0| < aC$ there exists unique $x \in [x_0 - a, x_0 + a]$ such that $y = f(x)$.

2. Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that for all $x, y \in [a, b] x > y$ we have $f(x) < f(y), \alpha := f(b), \beta := f(a)$.

a) Show that there exists a continuous function $g : [\alpha, \beta] \to [a, b]$ such that $g(f(x)) = x$ for all $x \in [x_0 - a, x_0 + a]$.

b) Assume that f is continuously differentiable. Show that g is continuously differentiable iff [=if and only if] $f'(x) < 0$ for all $x \in [x_0 - a, x_0 + a]$.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a continosly differentiable function such that $f(0, 0) = 0$ and $D_f(0, 0) \neq 0$ where $D_f(0, 0) : \mathbb{R}^2 \to \mathbb{R}$ be the differential of f at $(0, 0)$. Show that either you can find a continosly differentiable function $h : [-\epsilon, \epsilon] \to \mathbb{R}, h(0) = 0$ such that $f(h(y), y) = 0$ for all $y \in [-\epsilon, \epsilon]$ or you can find a continosly differentiable function $g : [-\epsilon, \epsilon] \to \mathbb{R}, g(0) = 0$ such that $f(x, g(x)) = 0$ for all $x \in [-\epsilon, \epsilon]$ or both.

The rest of the problems are from the chapter 9 in the book.

Section 1. Numbers 3,4,5

Section 4 Numbers 1,2,5