Chapter 12 Section 3 numbers 9,10,11,12

5)Let $sgn : \mathbb{R} \to \mathbb{R}$ be a function such that $sgn(a) = 1$ if $a > 0$, $sgn(a) = -1$ if $a < 0$ and $sgn(0) = 0$.

Find $\int_{x^2+y^2\leq 4} sgn(x^2 - y^2 + 2)dxdy$

6) a)Find determinants of the matrix

$$\begin{bmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{bmatrix}$$

b)Find determinants of the matrix

$$\begin{bmatrix} a & 0 & b & 0 \\ 0 & u & 0 & v \\ c & 0 & d & 0 \\ 0 & w & 0 & x \end{bmatrix}$$

and explain the answer.

7. Let $A = (a_{i,j})$ be an antisymmetric $n \times n$ matrix A [that is $[a_{i,j} = -a_{j,i}]$. Show that if n is odd then $Det(A) = 0$.

8.Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

Find $Det(A)$

A hint. Compute A^2

9. a)Let A be a 2×2 matrix $A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$. We define $\tilde{A} := \begin{bmatrix} a_{2,2} & -a_{1,2} \\ -a_{2,1} & a_{1,1} \end{bmatrix}$. Show that $A\tilde{A} = Det(A)Id$

b) Prove that a 2×2 matrix A is invertable iff $Det(A) \neq 0$.

For any $n \times n$ matrix A and any pair $(i, j), 1 \leq i, j \leq n$ we denote by $A_{i,j}$ an $(n-1) \times (n-1)$ matrix which is obtained from the $n \times n$ matrix A by removing $i - th$ row and $j - th$ column. We denote by $\tilde{A} = (\tilde{a}_{i,j}), 1 \leq i, j \leq n$ an $n \times n$ matrix such that $\tilde{a}_{i,j} := DetA_{j,i}$.

Remark. If $n = 2$ the definiton of he $n \times n$ matrix $\tilde{A}$ agrees with the definition from the part a).

c) Show that for $n = 3$ we have $A\tilde{A} = Det(A)Id$.

d) Is the equality $A\tilde{A} = Det(A)Id$ true for all n?

10. a) Check that for any a 2×2 matrix $A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$ such that $a_{1,1} \neq 0$ there exists unique real numbers x, y such that $A = B\Lambda C$ where

$B = \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} a_{1,1} & 0 \\ 0 & Det(A)/a_{1,1} \end{bmatrix}$ and $C = \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}$

b)Show that in the case when $a_{1,1} = 0, Det(A) \neq 0$ we can write A as a product $A = \hat{s}\Lambda C$ where $\hat{s} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Λ is a diagonal matrix and C is as in a)

c)Let $A(a_{i,j})$ be a 3×3 matrix such that $a_{1,1} \neq 0$ and $DetA_{3,3} \neq 0$. Show that we can write A as a product $A = B\Lambda C$ where B is a lower triangilar matrix with 1 on the diagonal, C is an upper triangilar matrix with 1 on the diagonal and Λ is a diagonal matrix.

d) Show that any 3×3 matrix A can be written as a product $A = B\hat{s}\Lambda C$ where B, Λ, C are as before and $s \in S_3$ [remember that $\hat{s}$ acts on $\mathbb{R}^3$ by $\hat{s}(e_i) := e_{s(i)}$].

A hint. Use results of 9.