1.a) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Show [by interchanging the order of integration] that for any $x > 0$ we have

$$\int_0^x (\int_0^s f(t)dt)ds = \int_0^x (x-t)f(t)dt$$

b)Find an expression for the triple

$$\int_0^x (\int_0^r (\int_0^s f(t)dt)ds)dr$$

in terms of a simple integral.

2. Use polar coordinates to evaluate the following integrals

a)

$$\int_B \frac{y}{x^2+y^2}dxdy$$

where B is the ring $1 \leq x^2 + y^2 \leq 4$

b)

$$\int_B \sin(x^2+y^2)dxdy$$

where B is the disk $x^2 + y^2 \leq 4$

3. Let B be the rectangle $a \leq x \leq b, c \leq y \leq d$

a)Let $f, g : B \to \mathbb{R}$ be continuously differentiable functions.

Evaluate $\int_B f_x(x,y) - g_y(x,y)dxdy$ where $f_x(x,y), g_y(x,y)$ are partial derivatives of the functions f and g.

Note that the result is in some sense an integral around the boundary of B.

b)Let $f : B \to \mathbb{R}$ be a twice continuously differentiable function. Show that

$$\int_B f_{xy}(x,y)dxdy = f(a,c) + f(b,d) - f(a,d) - f(b,c)$$

.

4.a) If $f(t) = \int_1^4 e^{tx^2}dx$ find $f'(t)$.

b)If $g(t) = \int_t^{t^2} e^{tx^2}dx$ find $g'(t)$.

c)Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous function

$$v(x,t) := \int_{2x-t}^{2x+t} g(s)ds$$

. Find v_x and v_t. Further, assuming that g is continuously differentiable show that $v_{xx} = 4v_{tt}$

5.Express the following iterated integrals as triple integrals and as iterated integrals for at least one other "order of integration"

a)$\int_{-1}^1 dy \int_)^{1-y^2} dx \int_{-\sqrt{x}}^{\sqrt{x}} f(x,y,z)$

b)$\int_0^1 dx \int)^1 dy \int_0^{x^2+y^2} f(x, y, z)$
6. a) Find the volume of the ball $x_1^2 + x_2^2 + x_3^2 + x_4^2 \leq a^2$
b)Generalize