# MATH 23B PROBLEM SET 2

*due February 18th*

Let $V$ be a vector space of diminsion $d$ and $e_1, ..., e_d$ a basis of $V$. We denote by $\mathcal{A}(r,d)$ be the set of subsets $I$ of $[1,...,d]$ such that $|I| = r$. For any $I = \{i_1, ..., i_r\} \in \mathcal{A}(r,d)$ we associate an $r$-linear form $\omega^I$ on $V$ by

$$\omega^I(v_1, ..., v_r) := \sum_{\sigma \in S_r} sign(\sigma) \prod_{1 \leq k \leq r} e^{i_{\sigma(k)}}(v_k)$$

1. a)Prove that $\omega^I \in \Omega^r(V)$.

b)Prove that the set $\omega^I, I \in \mathcal{A}(r,d)$ is a basis of the space $\Omega^r(V)$.

Let $A : V \to V$ be a linear map. We denote by $\Omega^r(A) : \Omega^r(V) \to \Omega^r(V)$ the linear map such that for any $\omega \in \Omega^r(V), v_1, ..., v_r \in V$ we have $\Omega^r(A)(\omega)(v_1, ..., v_r) = \omega(A(v_1), ..., A(v_r))$.

2. Prove that $\Omega^r(A \circ B) = \Omega^r(B) \circ \Omega^r(A)$

Since $e_i, 1 \leq i \leq d$ is a basis of $V$ there exists a $d \times d$-matrix $(a_{i,k})$ such that $A(e_k) = \sum_{1 \leq i \leq d} a_{i,k} e_i$. We call $(a_{i,k})$ *the matrix of an operator* $A$ in respect to a basis $e_i, 1 \leq i \leq d$.

3. Write the matrix of the operator $\Omega^r$ in respect to the basis $\omega^I, I \in \mathcal{A}(r,d)$.

A hint. First do the case when $r = 1$ and $r = 2$ ,then the case $r = d$ and only afterwords approach the general case.

4.a) Let $p(x,y)$ be a polynomial such that $p(0,y) = 0, \forall y \in \mathbb{R}$. Prove that $p(x,y)$ is divisible by $x$ (that is show the existence of a polynomial $q(x,y)$ such that $p(x,y) = xq(x,y)$.

b) Let $p(x,y)$ be a polynomial such that $p(y,y) = 0, \forall y \in \mathbb{R}$. Prove that $p(x,y)$ is divisible by $x - y$