1. Let V, W be vector space such that $dim(V) = 3, dim(W) = 2, e_1, e_2, e_3$ be a basis in V and l_1, l_2 a basis in W. Let $\omega \in \tilde{\Omega}^1_c(V)$ be a differential form given by $\omega(x, y, z) = xe^1 + ye^2 + ze^3, \phi : W \to V$ a map given by $\phi(al_1 + bl_2) = a^2e_1 + abe_2 + b^2e_3$. Find $\phi^*(\omega)$. [That is find functions f, g on W such that $\phi^*(\omega)(w) = f(w)l^1 + g(w)l^2$.]

2. Let $\omega^1, \omega^2 \in \tilde{\Omega}^1_c(\mathbb{R}^2)$ be differential forms such that $\omega^1(a, b) = e^1, \omega^2(a, b) = e^2$. Describe all smooth maps $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ such that $\phi^*(\omega^1) = \omega^1$ and $\phi^*(\omega^2) = \omega^2$.

3.a)Show the existence of a smooth [=infinitely differentiable] non-negative function g on $\mathbb{R}$ such that $g(x) = 0$ if $x < 0$ and $g(x) = 1$ if $x > 1$. [Use the results form the last homework in the Fall].

b)Show the existence of a smooth [=infinitely differentiable] nonnegative function f on $\mathbb{R}$ such that $f(x) = 0$ if $|x| > 1$ and $\int_{\mathbb{R}} f(x)dx = 1$.

Let f be as in the problem 3.For any $\epsilon > 0$ we denote by f_ϵ be the function on $\mathbb{R}$ such that $f_\epsilon(x) := 1/\epsilon f(x/\epsilon)$. For any continuous function F on $\mathbb{R}$ we define a function F^ϵ on $\mathbb{R}$ by $F^\epsilon(y) := \int_{\mathbb{R}} f_\epsilon(x)F(y-x)dx$

4.a)Show that the function F^ϵ is smooth.

Assume that there is a compact subset $C \in \mathbb{R}$ such that $F(x) = 0$ for $x \in \mathbb{R} - C$ [That is $supp(F) \subset C$].

b)Show that the function F^ϵ converge *uniformly* to F for $\epsilon \to 0$ [Please give a definition of the uniform convergence].

For any point $z \in \mathbb{R}$ we define $d(z, C) := min\{|x - z|\}x \in C$.

c)Show that for any $z \in \mathbb{R}$ such that $(z, C) > \epsilon$ we have $F^\epsilon(z) = 0$.

d)Formulate and prove the analogous result for functions on $\mathbb{R}^n$.

5.a) Let $g(x) := exp(-x^2)$ Show that for any integer $n > 0$ we have

$$\int_{\mathbb{R}} g^n(x)dx = \sqrt{\pi/n}$$

b)Let f be a smooth non-negative function on $[-1, 1]$ such that $f(0) = 1, f(x) < 1$ for $x \neq 0$ and $f''(0) \neq 0$. Let $I_n := \int_{-1}^1 f^n(x)dx$. Prove that

$$lim_{n\to\infty}\sqrt{n}I_n = \sqrt{2\pi}/\sqrt{-f''(0)}$$