MATH 23b, SPRING 2002
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Homework Assignment # 5
Due: March 15, 2002

Homework Assignment #5 (Final Version)

1. Reading:
   - Shilov Sections 7.2–7.5 and 10.1–10.2.
   - Edwards Section 2.8.

2. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be of class C^3. Let $\mathbf{a}$ be a critical point of f, that is, $\nabla f(\mathbf{a}) = \mathbf{0}$. Show that if the quadratic form $q(\mathbf{h})$ corresponding to f at $\mathbf{a}$ is positive-definite, then $f(\mathbf{a})$ is a local minimum.

   (We are looking for a δ-ε proof using the equation:

   $$f(\mathbf{a}+\mathbf{h}) - f(\mathbf{a}) = q(\mathbf{h}) + R_2(\mathbf{h}).$$

   You may use Taylor's Theorem without proof.)

3. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be given by

   $$f(x, y, z) = x^2 + xy + z^2 - \cos y.$$

   (a) Show that $\mathbf{0}$ is a critical point of f.

   (b) Find the quadratic form q of f at $\mathbf{0}$, and the associated matrix A.

   (c) Find the eigenvalues of A and the associated orthonormal eigenbasis for $\mathbb{R}^3$.

   (d) Determine the nature of the critical point $\mathbf{0}$ from a consideration of the eigenvalues of A.

4. Taken from Edwards p. 158, problem #8.2 and 8.3:

   Let $q(x, y, z) = 2x^2 + 5y^2 + 2z^2 + 2xz$ be a quadratic form. Show that q is positive-definite by:

   (a) using Theorem 8.8.

   (b) diagonalizing the quadratic form.

5. Find the minimum value of the function $f(x, y, z) = x^3 + y^3 + z^3$ on the intersection of the planes $x + y + z = 2$ and $x + y - z = 3$.

6. Finish the example from class on Wednesday, March 13!

   That is, consider the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ given by:

$$f(x, y) = x + x^2 + xy + y^3$$

   (a) Find all critical points of f by setting $\nabla f = 0$.
   (Hint: There are two.)

   (b) At each critical point of f, find the quadratic form, the associated matrix, and its eigenvalues.

   (c) Classify the "definiteness" of the quadratic form of f at each critical point, and identify each critical point as a local maximum, minimum, or neither.