MATH 23b, SPRING 2003
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Homework Assignment # 10
Due: May 2, 2003

Homework Assignment #10 (Final Version)

1. Read Fitzpatrick, Chapter 19.
2. (A) Let $F : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be given by $F(x, y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$.

   Let C_0 be the unit circle in $\mathbb{R}^2$ parametrized by the function $\gamma_0 : [0, 1] \longrightarrow \mathbb{R}^2$, where $\gamma_0(t) = (\cos 2\pi t, \sin 2\pi t)$.

   Let C_1 be the circle of radius 1 centered at $(1, 1)$ in $\mathbb{R}^2$ parametrized by the function $\gamma_1 : [0, 1] \longrightarrow \mathbb{R}^2$, where $\gamma_1(t) = (1 + \cos 2\pi t, 1 + \sin 2\pi t)$.

   (a) Show that $\int_{C_0} F = 2\pi$.

   (b) Show that $\int_{C_1} F = 0$.
3. (B) Let $F(x, y, z) = (z^3 + 2xy, x^2 + 1, 3xz^2)$ be a vector field on $\mathbb{R}^3$. Show that F is conservative by computing the partial derivatives of its component functions, and find $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ such that $F = \nabla f$.
4. (C) Evaluate $\int_C 2xyz\, dx + x^2 z\, dy + x^2 y\, dz$, where C is a piece-wise smooth oriented curve from $(1, 1, 1)$ to $(1, 2, 4)$.