MATH 23a, FALL 2002
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
(Final Version) Homework Assignment # 2
Due: October 4, 2002

1. Read Chapter 1 and Section 2.1 of Fitzpatrick, and Sections 3 and 4 from Chapter 2 of Curtis.

2. (A) Verify that the operations addition and multiplication are well-defined for rational numbers, as defined by equivalence classes of ordered pairs of integers.

3. (A) For the fields $F = \mathbb{Q}$ and $F = \mathbb{R}$, prove the Triangle Inequality:

$$|a + b| \leq |a| + |b|, \forall a, b \in F$$

   (Hint: This is meant to be a single proof that works in both cases. You do not need equivalence classes, just the set P of positive elements.)

4. (B) Prove that $\sqrt[3]{2}$ is an irrational real number. (In other words, show that there is a real number x that satisfies the equation $x^3 - 2 = 0$, but that x is not rational.)

5. (C) Considering the real numbers as defined by equivalence classes of Cauchy sequences of rational numbers, name the equivalence class that acts as the multiplicative identity, and verify that it does.

6. (C) Considering the real numbers as defined by equivalence classes of Cauchy sequences of rational numbers, prove the existence of multiplicative inverses (for elements other than the additive identity).

7. (D) Use the definition of Cauchy sequence to show that the sequence of rational numbers $\left\{ \dfrac{1}{n^2} \right\}_{n=1}^{\infty}$ is a Cauchy sequence.

8. (D) Show that $\mathbb{N}$ is unbounded (above) as a subset of $\mathbb{R}$.

   (Recall that the natural numbers $\mathbb{N}$ satisfy the Peano axioms. The important one as far as this problem is concerned is that if $n \in \mathbb{N}$, then $n + 1 \in \mathbb{N}$.)