Math 23a, 2002.
Solution Set 9, Question 4.
Joshua Reyes

Question 4. Let $\dim(V) = n$, and let $f : V^k \to F$ be an alternating k-linear form with $k < n$. Show by example that it is possible to have a set of k linearly independent vectors $\{v_1, \ldots, v_k\}$ in V such that $f(v_1, \ldots, v_k) = 0$. (Make sure that $k \geq 2$ so that f can be alternating!)

Answer. Tons of people gave me tons of answers. Here's one:
Let $V = \mathbb{R}^3$ and consider $f : V^2 \to \mathbb{R}$ where

$$f(v, w) = \det \begin{bmatrix} 1 & v_1 & w_1 \\ 1 & v_2 & w_2 \\ 1 & v_3 & w_3 \end{bmatrix}.$$

Now, we know that f is alternating and bilinear because $\det : \mathbb{R}^3 \to \mathbb{R}$ is alternating and multilinear in its columns. And it's totally true that the vectors $v = [1 \quad 0 \quad 0]^t$ and $w = [0 \quad 1 \quad 1]^t$ are linearly independent (because they're orthogonal, etc.); however

$$f(v, w) = \det \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 0.$$

This sort of thing generalises in a nice way to an $(n-1)$-linear form. Just take the determinant of $n - 1$ vectors with something silly like the vector of all ones in the first column.