

MATH 23b, SPRING 2003
THEORETICAL LINEAR ALGEBRA
AND MULTIVARIABLE CALCULUS
Homework Assignment # 6B
Would Have Been Due: March 21, 2003

Homework Assignment #6B (Moral Homework)

1. Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be given by

$$f(x, y, z) = x^2 + xy + z^2 - \cos y.$$

- (a) Show that $\mathbf{0}$ is a critical point of f .
 - (b) Find the quadratic form q of f at $\mathbf{0}$, and the associated matrix A .
 - (c) Find the eigenvalues of A and the associated orthonormal eigenbasis for \mathbb{R}^3 .
 - (d) Determine the nature of the critical point $\mathbf{0}$ from a consideration of the eigenvalues of A .
2. Let $q(x, y, z) = 2x^2 + 5y^2 + 2z^2 + 2xz$ be a quadratic form. Show that q is positive-definite by diagonalizing the quadratic form.
3. Find the minimum value of the function $f(x, y, z) = x^3 + y^3 + z^3$ on the intersection of the planes $x + y + z = 2$ and $x + y - z = 3$.
4. Finish the example from class on Wednesday, March 12!

That is, consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by:

$$f(x, y) = x + x^2 + xy + y^3$$

- (a) Find all critical points of f by setting $\nabla f = 0$.
(*Hint: There are two.*)
- (b) At each critical point of f , find the quadratic form, the associated matrix, and its eigenvalues.
- (c) Classify the “definiteness” of the quadratic form of f at each critical point, and identify each critical point as a local maximum, minimum, or neither.